Conservatorium van Amsterdam

AN ORGANIZING TIDE OF CHAOS

Resonances of Chaos Theory in a Composer's Craftsmanship

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To Paulina,

an invaluable companion in my

artistic/intellectual quest.

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I would like to thank the following people for their influence on this work through their ideas, advise, belief, criticism, support, and/or patience: Alejandro, Alejandro, André, Armando, Arturo, Catalina, Carlos, Christer, Diego, Dolores, Fabio, Ingrid, Iñigo, José, José, Jorge, Lolke, María Antonieta, Michiel, Odette, Odette, Peter, Renato, Ruy, Rodrigo, Víctor and Wim.

Contents

I. **Introduction**

A high organization of musical material has been one of the central preoccupations of several musical movements throughout the XXth century. Amongst the most notorious composers who have followed this path we find Webern and Berg, two central figures of the serialist movement; Ligeti, with his use of mechanisms and golden proportion; and structuralists such as Boulez or Donatoni.

On the other hand, during the same period, there have also been musical movements that point in an apparently opposite direction. Such is the case of composers like Cage, who recurred to aleatory compositional processes (in the use of the *I Ching*, for example, or in the copying of imperfections from different kinds of papers on to manuscript paper); and of Xenakis, who recurred to probabilistic and set theories to develop his music.

Nonetheless, when it comes to the musical results, both tendencies have, in many cases, arrived at a quite similar outcome. One can think for instance of a dense orchestral texture. Ligeti arrived at this result by mechanical extreme organization of contrapuntistic material (in *Atmospheres* for example); while Xenakis achieved such textures (which sound quite different from Ligeti's) by the use of the Maxwell-Boltzmann Law (a law that describes the general behaviour of spreading gases, while the description of each gas particle remains impossible) in *Pithoprakta*.

Such concerns in these composers' aesthetical views have, without a doubt, been informed or even inspired by scientific and/or philosophical concepts that were central to their lives (Christianity, relativity, Buddhism, probability). Furthermore, if a common ground amongst these divergent composers can be found, perhaps the common denominator, associated to their historical origin in Schöenberg's break-up with tonality, is their formalization of an extra-musical motivation into their personal musical language. Such formalization became a central concern of most composers who, given the break-up with the paradigm of tonality, found themselves in need to develop new frame-works for musical composition. Account of such formalization can be found, for instance, in Messiaen's, Xenakis', Cage's or Boulez's writings, to mention a few.

As an active composer at the beginning of my creative career, I find it important to follow such a formalizing path as an adequate means to project my extra-musical ideas into my craftsmanship. Particularly, the extra-musical motivation that I find relevant to incorporate into my artistic doing is Chaos Theory.

At the mere construction level, I find such formalization especially relevant given the fact that music, as a temporal art, and due to its abstract nature (where form and content are one and the same thing), allows the exploration of one of the central concepts of Chaos Theory: the deterministic behaviour of chaos. Such behaviour implies that chaos and order are, at a functional level, one and the same thing, regardless the fact that they are perceived as opposite. Since this is a core concept to how I interpret both the natural and the social domains, in other words, how I perceive reality; the inclusion of such a concept into my musical doing is not only a logical, but also an ethical, step.

Furthermore, I also find it central to give an account of how I see such ideas already present in composers that followed a different formalization process and, that had already exerted a strong influence in my musical language.

The present work is a first systematic effort in the mentioned direction. The structure of which is as follows: I begin with a chapter that links Donatoni's Music with fractals; then I go into a chapter that exposes the beginning of an on-going development of a musical material based on the concept of Chaos Theory and; finally, in a third chapter, I show how the two preceding chapters influence one of my compositions.

II. **Donatoni's *Françoise Variationen I, II and III*: Self-Similarity and Change**

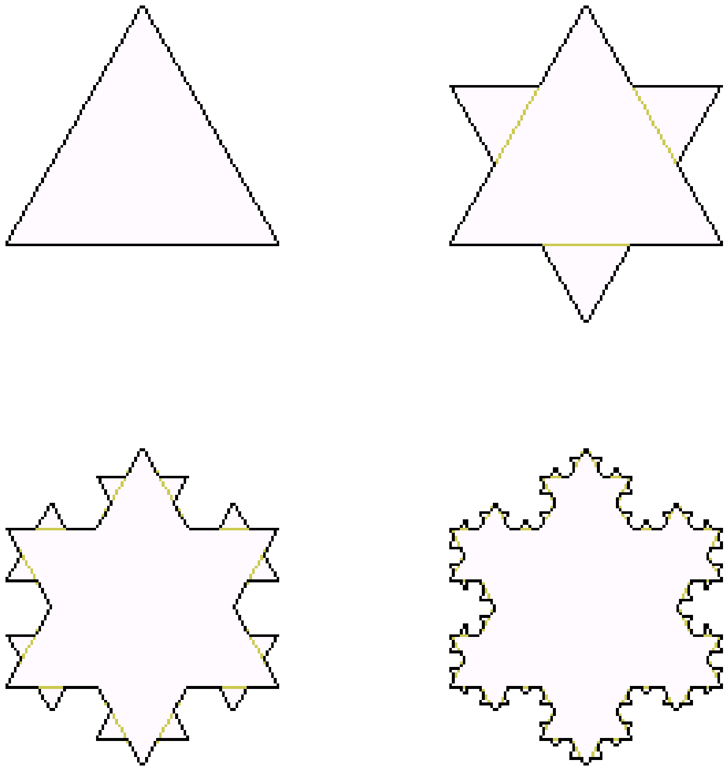
Perhaps Franco Donatoni's most fascinating compositional technique is his method of re-reading his own previous musical production.[1] This method proved fruitful to such an extent, that the composer reported working exclusively with it for over a decade. The *Françoise Variationen* is an excellent example of such a procedure. The work consists of seven sets of seven variations each (forty-nine variations in total). With the exception of the first variation, which was derived from one of his previous works, every other variation is derived by means of re-reading its predecessor.

As can be intuited, underlying the *Françoise Variationen* there are two common denominators. Firstly, the persistence of the material and, secondly, the transformation of this material by means of a new methodological procedure rather than by the introduction of new elements. These two common denominators coincide with those underlying the principle of fractal development. Take as an example on of the simplest fractals: Koch's Curve.

Such a fractal, also known as "snow flake", can be constructed from an equilateral triangle. Each of the triangle's sides is divided into three equal segments. From the points that divide each of the lines, and using the middle segments of each of the sides of the original triangle, three equilateral triangles (who's side's length is one third of the original triangle's side) is constructed. This leaves us with a twelve equal-sided star. Each of the star's sides is divided into three equal segments and, from each middle segment, a new equilateral triangle is drawn, just as in the previous step. Such a method is iterated over and over (see diagram below).[2]

[1] I employ "re-reading" as a literal translation of the Spanish *relectura*; term employed by one of Donatoni's most renown former students, Víctor Rasgado. The term implies both a second reading and a re-interpretation of a given original.

[2] For a detailed rendering of this example see: Elizer Braun, *Caos, Fractales y Cosas Raras*. Fondo de Cultura Económica, Mexico City, 1996. pp. 23-26.



Note how, after each stage of the constructing procedure, the original figure and sole constructing element, the equilateral triangle, while always present, becomes less and less evident as the iterations are carried on. Hence, a variety that coexists with a self-similarity is achieved. Such an interrelation happens in fractals not only within the same scale but is independent of scale. Just like, for instance, the patterns that can be observed on a tree's leaf are similar to the tree itself (or to another of Nature's constructions: the human lungs).[3]

Something quite similar occurs with Donatoni's procedure. Let us look at the method of construction. Allow me to ignore for the moment (I will com back to it later) the fact that each variation is not constructed from a unique method but, derived from the previous one; and to focus on what is unique in the construction: the method of re-reading the previous material.

The construction of each variation can be explained, and recreated, by the use of a single algorithm per variation. After re-analysing the variations, I have designed algorithms that, parting from the previous variations, reproduce the subsequent one.[4]

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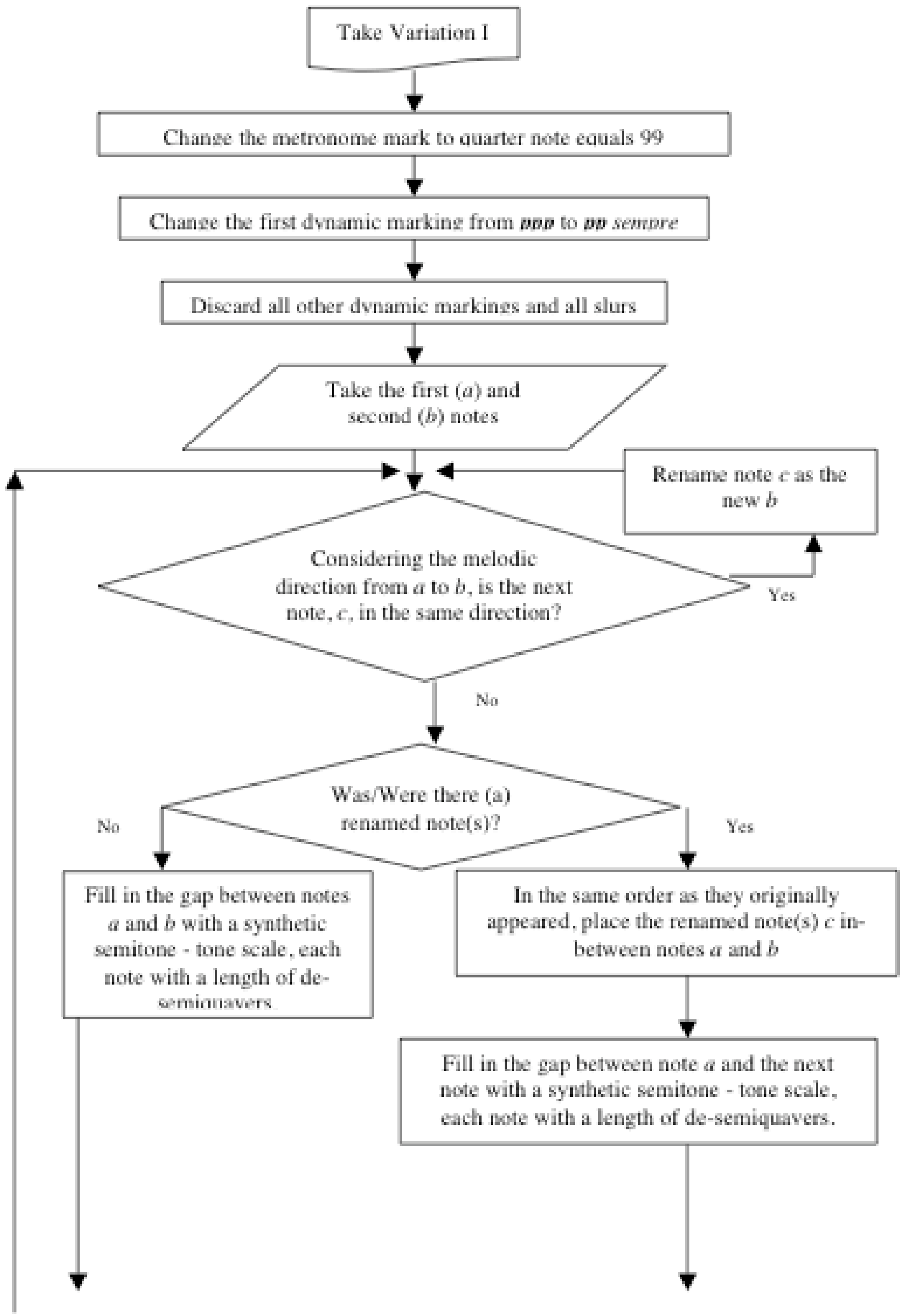
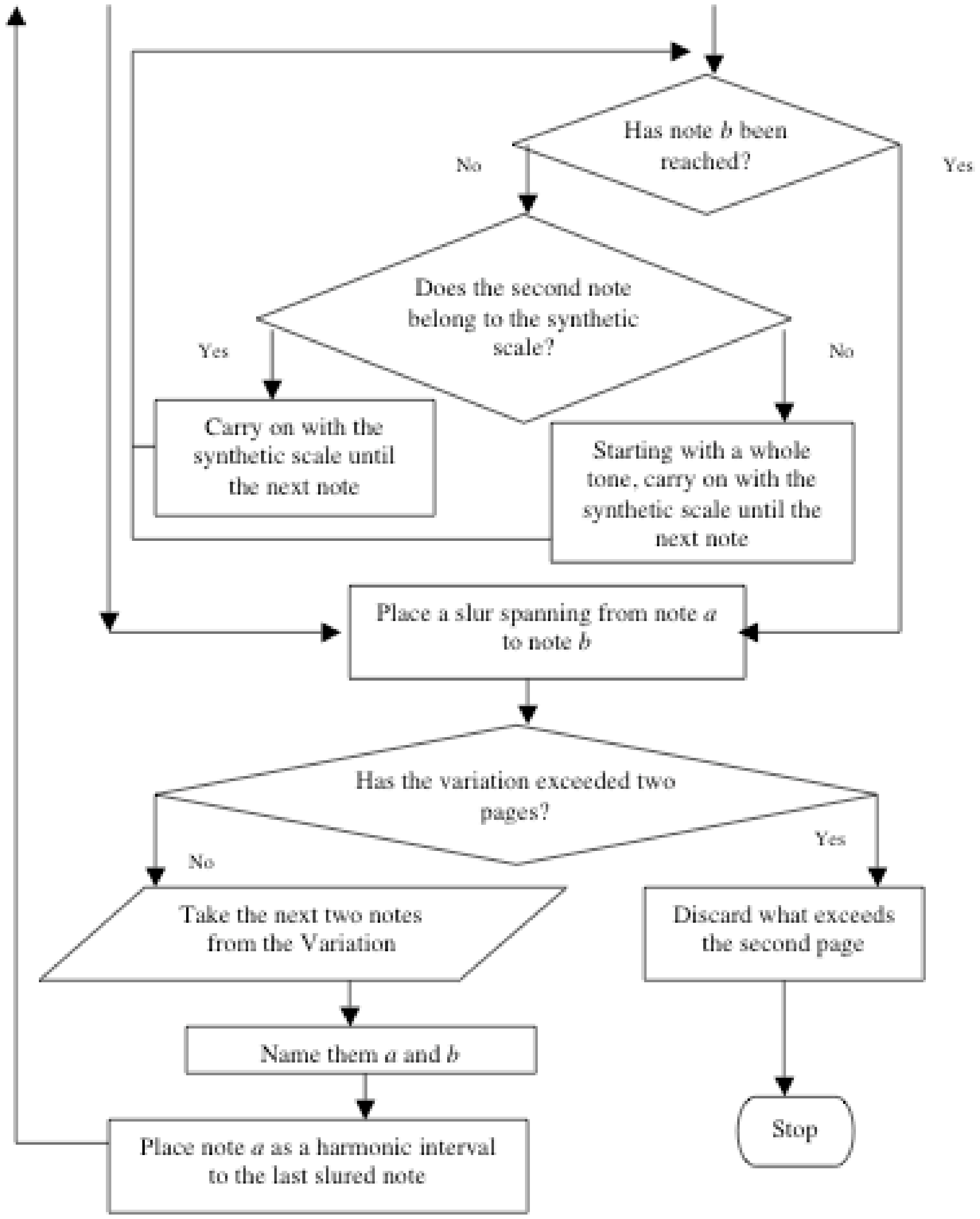
[3] Within this text, capitalized "Nature" refers to "the physical power causing all the phenomena of the material world" Della Thompson (ed.), *The Concise Oxford Dictionary*. Oxford University Press, London, 1996.

[4] The original analysis was carried out in Víctor Rasgado's Analysis and Composition Workshop; the development of the algorithms is mine.

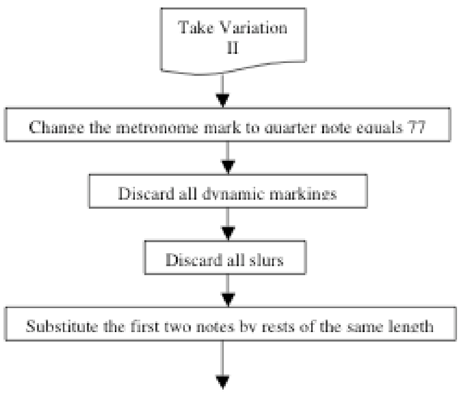
Let us then look at the sequence of instructions that reproduce the second and third variations:

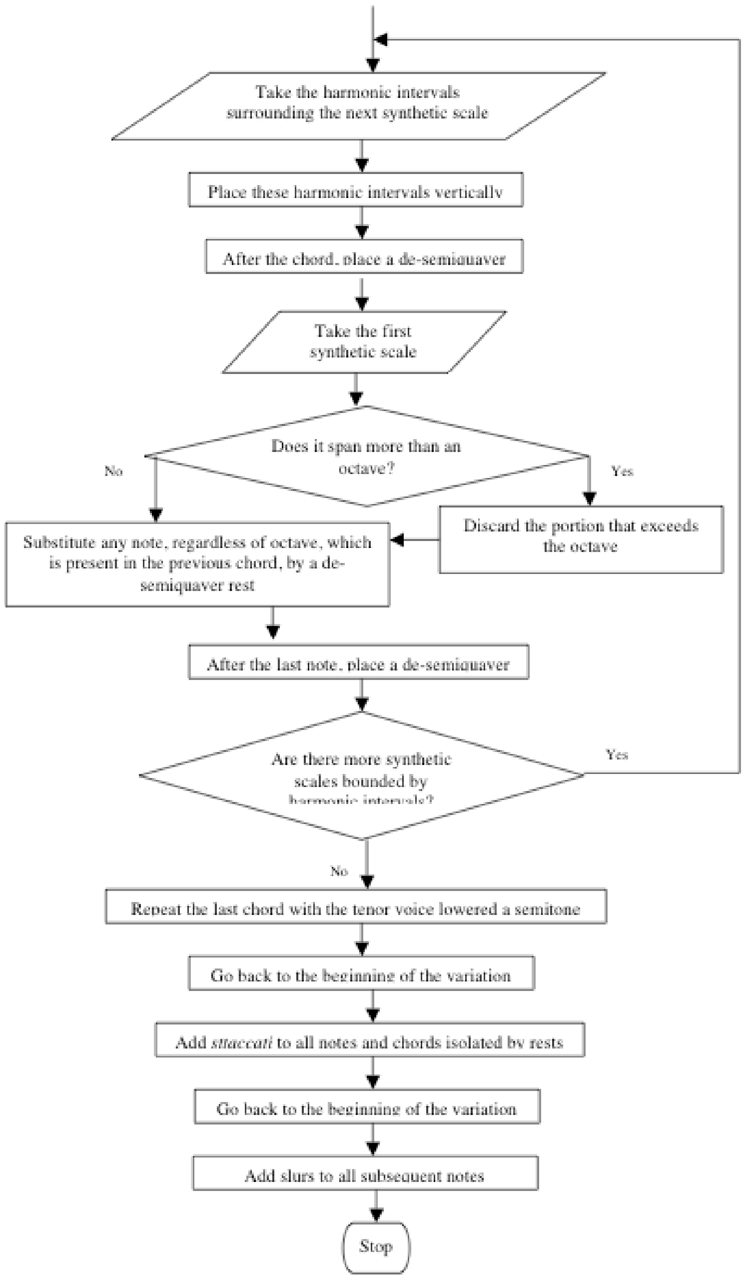
Algorithm for Deriving Variation II:





Algorithm for Deriving Variation III:





Before going on with the discussion, it is important to point out that while these flowcharts give instructions to reconstruct the relevant variations, they do not exactly reconstruct them. There are two reasons for this: the first one is that, for clarity's sake, no instructions were given concerning note spelling or voicing; and the second one is the fact that there are five deviations from the re-reading method.[5]

[5] The mentioned deviations are: 1) in the second system of the second variation, between the second and third phrases, a rest appears; 2) in the fourth system of the same movement, the first two notes were expected to be placed simultaneously, but they are not; 3) in the last system of variation II, between the third and fourth phrases, a rest appears; 4) in the fifth system of the third variation, on the third chord, pitches G# and

This said, let me introduce the core idea under which it is proposed that each pair of variations have, through self-similarity and change, fractal characteristics. Such proposal rests on the concept of contingency. Although Koch's Curve and more complex 'lab' fractals, like Mandelbrot's Universe, were conceived as an effort to measure and understand some of the world's phenomena more accurately than with Euclidean geometry (through a shift from a quantitative to a qualitative approach), it is important to point out that these fractals are still not how Nature behaves. Perhaps the most stunning characteristic of Nature, and life particularly, is its ability to adapt itself to unforeseen events and, moreover, make use of them. Take for instance a comparison between a computer and the human mind: when learning how to program a machine it is common to get an error message that reads "array at line *x*".[6] This message is actually generated by another program that prevents the machine from trying to solve a self-referent argument that would make the system crash. However, a similar kind of self-reference is used in Zen Buddhism in order to help seek illumination. This is possible because self-referent arguments have a completely different effect on the mind than on a computer.[7]

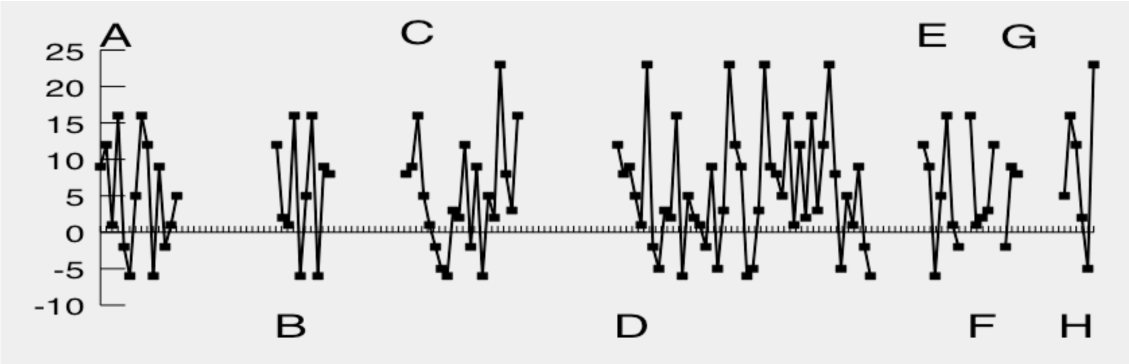
What I am proposing is that while dealing with contingencies (which in this case take the form of the variation from which to develop and of human mistake) the *Françoise Variationen* resemble real conditions more than creating a self-enclosed system that, like Euclidean geometry, is more conceptually round, but, in fact, quite detached from reality.

Let us postpone this argument until the conclusion and, for now, deal with similarity, change and the different scales where these occur. Perhaps the clearest similarity between the relevant variations appears between the contour line of variations I and II. In order to make such a comparison clearer, I designed graphical representations of the pitch activity against time. Let us first look at the graph that represents the pitch-contour information of Variation 1:

C are expected to appear but they appear until the next chord; at the same time on the latter chord, F and A are expected but they appear on the previous chord and; 5) in the seventh system of the third variation, between the third and the fourth chords, the same kind of substitution takes place (this time, the expected succession that is inverted is C# and C, which are interchanged with D and E). An objective explanation for such deviations could not be found.

[6] Where *x* can be any line in the program.

[7] *Cf.* Phillip Kapleau, *Zen, Dawn in the West.* Anchor, New York, 1979.

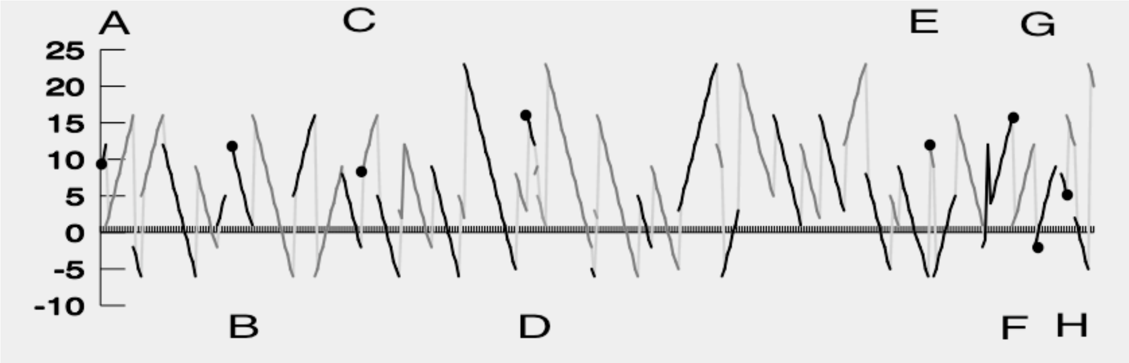


Some explanation on the data displayed: the y-axis represents the pitches in their corresponding register;[8] the x-axis represents a time distribution expressed in de-semiquavers; the dots represent the variation's pitch information from the beginning until the last note employed to construct Variation II;[9] the lines that join points represent the contour formed by consecutive notes (i.e. not separated by rests); the gaps between contours represent rests; and the letters above and below the graph represent comparison points that will be used later on.

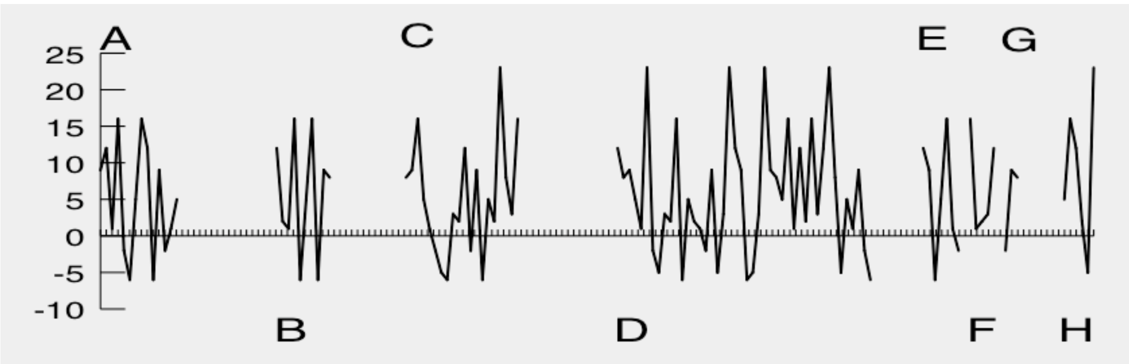
This said, let us look at the graph of the whole Variation II:

As can be seen, there are similarities in both graphs. However, the contour lines on the second variation are incomplete. This is due to the fact that the second variation is written mostly in two voices – a third one appears seldom – which only play simultaneously at the first and last note of each other. In order to get a clearer representation of the contours, I wrote a third "voice" which simply merges the original voices. Let us look at the result (for clarity's sake, I erased the points representing the pitches):

[8] Zero represents central C and ascending and descending semi-tones correspond to the natural numbers.
[9] This last note is the sixth B that appears in Variation I.



A word on the information displayed: the darkest lines represent the contour of voice 1; the intermediate lines represent voice 2's contour; the lightest line accounts for the general contour; and the newly added dots represent the starting point of the segments (marked by capital letters), which will be used to compare the contours of the three variations. Let us confront the previous image with the contour representation of Variation I (this time without the pitch dots):



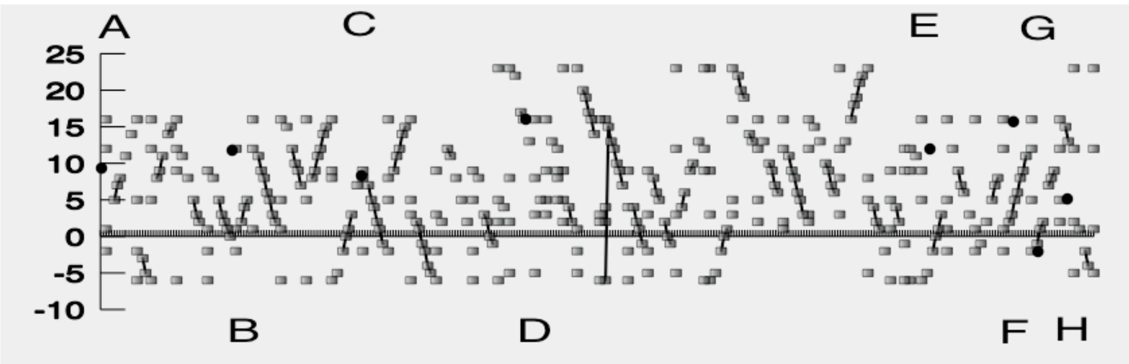
The first thing that must be said is that, although the scale of the y-axis is preserved, the scale of the x-axis is quite different. In both graphs the lines over the latter axis represent de-semiquavers. These marks can be seen clearly on the contour graph of Variation I. However, the scale is so much smaller for Variation II (less than one half of the one used to represent the equivalent segment of Variation I) that such markings are seen as a quasi-solid line. In addition, the de-semiquaver scales are not comparable since each variation requires a different *tempo* for its execution.[10] Secondly, it is important to note that if one compares any equivalent segment (A, B, C, etc.)

[10] It is interesting to point out that the mentioned *tempi*, crotchet equal to 88, 99, and 77, for variations I, II and III, respectively, all have eleven as a common factor. This fact is not dealt with thoroughly since, in order to understand the underlying logic, the next variations must be taken into account.

between both graphs, they have the following similarities: the same numbers of 'peaks' and 'valleys', such amplitudes have identical magnitudes in both graphs and, the slopes between the analogue segments are also comparable. Of course, there are also differences but these will be dealt with later.

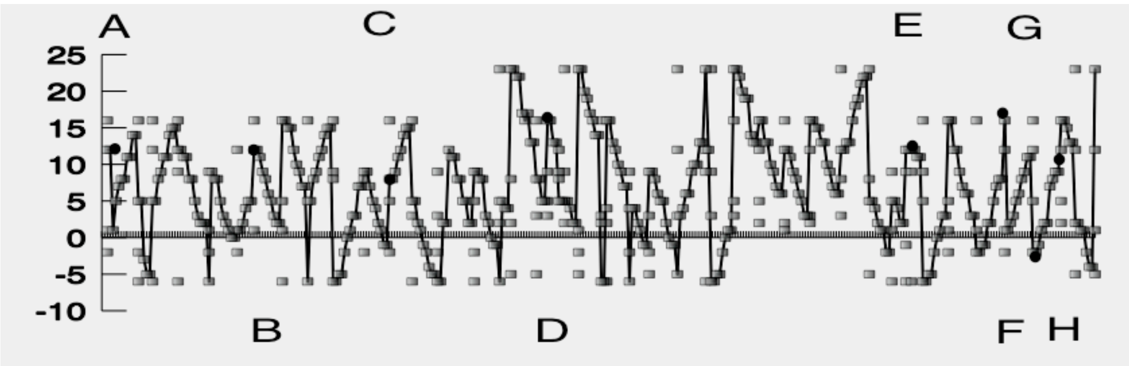
This resemblance accounts for one of the properties common to all fractals: similarity across scales. Take for instance the figure that depicts the construction of Koch's Curve (see page 4). Although the final shape is quite different to an equilateral triangle, the latter is present both implicitly (it is the sole building block of the curve) and explicitly in a much smaller scale (on the edge of the fractal).

While we are still dealing with similarity, it becomes relevant to find out how similar an analogue graph of Variation III would be. Since it was derived from Variation II, one can already intuit that it will be similar to its direct predecessor. But, how about its possible similarity with Variation I? It is important to point out that different re-reading methods were used to derive Variation II from Variation I, and Variation III from Variation II. This is a central difference with how fractals are constructed: a same function is iterated to construct a fractal; while Donatoni used different methods (functions) to derive each new part. Lets take a look at what the new graph (for Variation III) looks like:

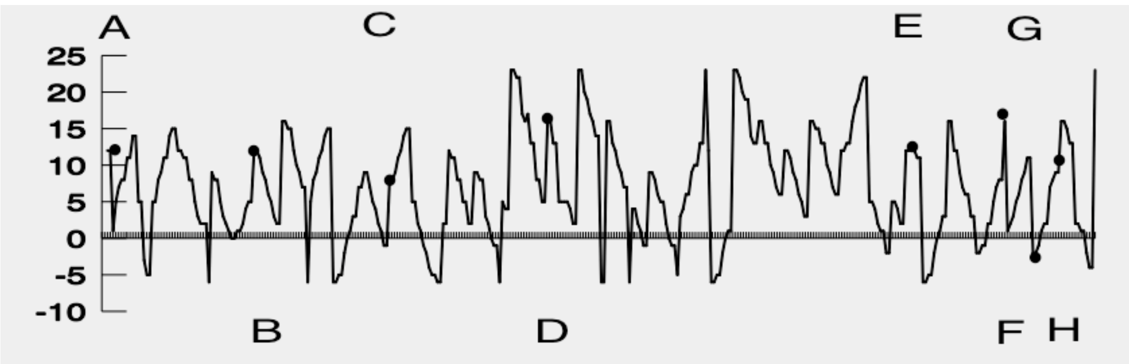


At first glance, it is quite obvious that there are deep similarities with Variation II. If it were not for the difference in *tempo* markings, the scale would be identical. However, it is impossible to say that the contour lines are similar. Just by looking at the amount of pitches present at both limits of the register this possibility can be discarded. Nonetheless, I am still thinking of Koch's Curve and how, at first glance, the originating triangle does not seem to be present after several iterations of the development process. Is the original contour (of Variation I) still present inside Variation III, regardless of the fact that Donatoni did not use the same method to develop parts II and III? My intuition tells me they are, but it is not obvious by looking at the graphs.

In order to answer my question, I had to deal with one problem: Since the variation is full of rests and, in order to show rests graphically, I chose not to draw contour lines over rests, I had to substitute the rests with pitch information that would affect the qualitative characteristics of the graph as little as possible. I chose to, whenever I came across a rest, replace it with the same pitch that preceded it (the other option I had was to suppress rests, but this would have altered a parameter that I wished to keep constant: the relative de-semiquaver distribution). The graphical result of the latter decision is as follows:

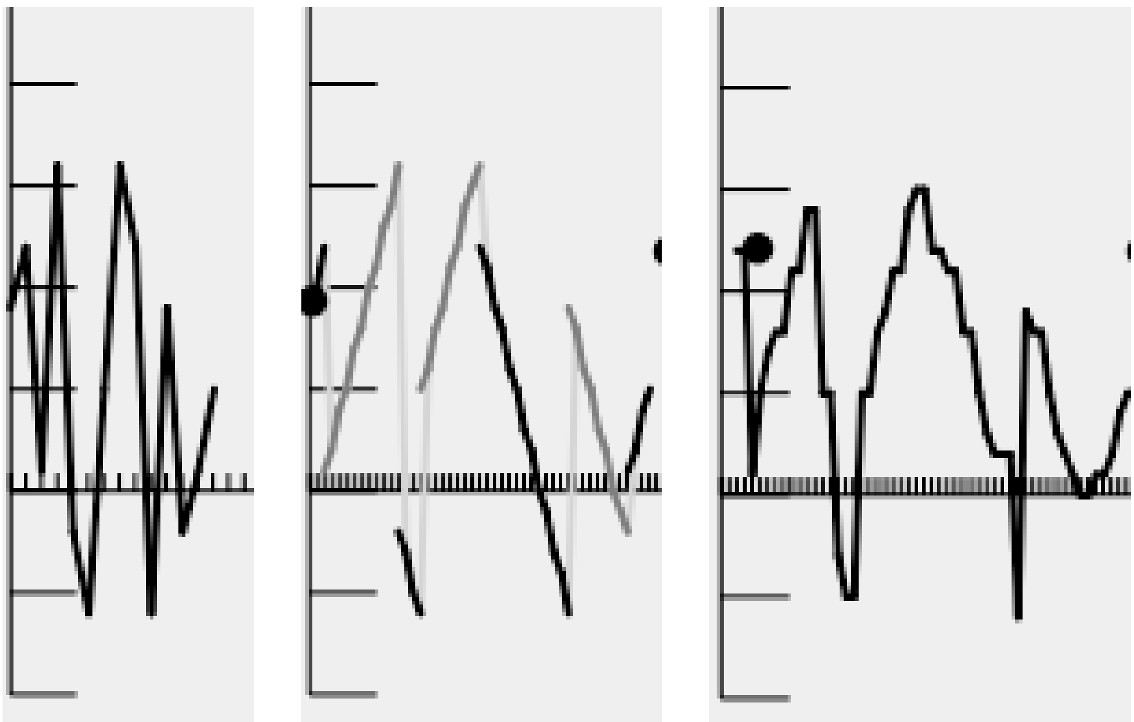


Again, for clarity's sake, I filtered out the points representing pitches and kept the resulting contour lines (note that this time such a process does cause some data loss: the harmonic intervals, but brings out-front the information I am looking for):



As can be seen, despite the fact that the third variation was derived from the second one, and despite the fact that the re-reading logic from the first variation to the second one is different from the method used between variations II and III, the backbone of Variation III is still Variation I. What calls my attention concerning this outcome is that, although Donatoni did not iterate the same function, but changed the process quite drastically (the first method adds pitches and disappears rests while the second one tends to create rests), he is arriving at an analogue result by a contradictory method.[11] Furthermore, each variation revolves around a different, easily distinguishable and particular gesture. Each gesture is constituted by a self-similar motivation and is also quite different to the other gestures but, nonetheless, they all conserve a same basic structure.

Regardless of such structure, each of the relevant variations has a singular gesture namely: 1) pointillist monophonic phrases; 2) scales and; 3) chords, short scales and single notes, for variations I, II and III, respectively. These explicit changes are easily explained by the different re-reading methods employed, which, in fact, reflect a conscious search for variety. However, within the context of this text, I find it more relevant to look at the quantitative changes within the qualitative stasis of what I've called the backbone structure. In order to do so, I reproduce a detail of segment "A" from the three variations' backbone contours:



The leftmost diagram shows the contour of the first phrase of Variation I. If we compare it to its equivalent in Variation II (centre diagram), we can observe that, although the outline is quite similar, some of the lines' gradients are flatter than in the

[11] The title of the work tends to point out that this was no accident.

first graph. In fact only the lighter grey lines, the artificial ones (remember that they were added to preserve a continuous contour), remain identical. Furthermore, on the excerpt from Variation II, the real components of the contour's slope (black and dark grey lines), become flatter. Additionally, the lines depicting it observe a regular wavy pattern. This is due to the fact that in the gaps existing in Variation I were, by means of the re-reading method, filled by octatonic scales.

Now, if we look at the contour segment taken from Variation III (rightmost diagram), we can still observe the same contour that the previous two variations exhibited. Nonetheless, it can also be seen that its outline possesses the greatest degree of distortion. Excepting for the first upward slope present in variations I and II, the peaks and valleys of the previous contours persist. However, the lines that arrive at these extremes become quite irregular. On the other hand, such irregularities are, paradoxically, constructed from the same material that gave shape to the previous contours: jumps (as in the Variation I) and octatonic scales (as in Variation II).

Nevertheless, the similarity of the already compared contours is clear. As a result, a very special quality can be observed throughout the three variations: the formal nature of Variation I is preserved as structural material for the second variation.[12] This statement is also true between the second and third movements of the piece, only that, in the third variation, both preceding movements are present as structure. At the same time, we cannot speak about a persistence of form, since each variation possesses a characteristic and distinctive gesture. It must be said that such an overlaying of structural information, by applying different methods on distinct (though related) material, is only possible thanks to Donatoni's mastery of re-reading. This is due to the fact that the sole decision of a re-reading method can be enough to derive a new piece out of an existing one. But, in order to preserve such a structural similarity, a deep understanding and command of the technique, plus an adequate choosing of the original material, are necessary. This method is, with respect to the ongoing comparison to fractals, a quite different way in which to achieve both self-similarity and change.

While it has not been claimed that the analysed *Françoise Variationen* are fractals, an analogy to the latter has been made. Furthermore, such a model directly inspired the present text's conception. Criticism to such an analogy may be easily raised by arguing that, in fact, the analysed music may be analogized to fractals only if one compares certain scales.

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[12] I make use of the terms "structure" and "form" as defined by John Cage, *Silence*. Wesleyan University Press, Hanover, 1973. p. 62

As a response, it can be said that there are no known strict fractals in Nature. Just as Euclidian geometry, fractal geometry is a model that serves the purpose of approaching phenomena. For instance, while strict fractals exhibit similarity regardless of scale, such a characteristic exists in Nature-constructed fractal-like objects over an extended, though limited, dimension range. To illustrate this one can think of broccoli: while at eyesight such a vegetable exhibits similarity across scales, at a cellular level, this resemblance is no longer observable. In the same way, the relevant variations exhibit a close structural resemblance but, only when observed through certain different (time) scales.

While comparing the limits of fractal-like behaviour in Nature to the limits of structural similarity in Donatoni, it becomes relevant to make reference to a previous observation: the deviations from the re-reading method. It can be argued that such departures are mistakes. If a computer composed the variations following the criteria dictated by the previously presented algorithms, then there would be no question that such deviations would be errors. However, if such music were composed by Nature, then the algorithms would be, instead of judging parameters, partial explanations for the phenomena. These changes that can be observed with respect to an abstraction model of the variations, whether by conviction or omission, seem to be compositional decisions.

In any case, such contingencies seem insignificant to deal with when compared to how Donatoni deals with another more determining situation: his previously composed work. In contrast to how fractals are constructed, through a singular method (whether deterministic or stochastic), Donatoni obtains both similarity and change by, from variation to variation, substituting the functions (re-reading procedures) that serve as his construction methods. Just like life, he adapts to the given conditions. Adaptation in Donatoni's work implies both change (in method and form) and, through self-similarity, continuity (in style and structure).

Through that process of adaptation he changes (in method and form) and also, through self-similarity, maintains a continuum (in style and structure).

III. **Towards a Chaotic Musical Material**

If one is willing to write music that represents chaos, then it makes sense to do so by employing a musical material that is chaotic itself. When someone hears such a term, the usual meaning associated to it is that of a total lack of order.

However, lack of order, i.e. disorder, is neither a valid route to achieve an aesthetically valuable goal, nor is it the connotation of the term “chaos” with which I intend to deal. Therefore, it is central to the good understanding of this text to clarify the usage of the mentioned term: the definition of “chaos” which from now on I will employ is the one enclosed in the discourse of Chaos Theory which can be defined as follows:

> Chaos Theory (noun) Describes the complex and unpredictable motion or dynamics of systems that are sensitive to their initial conditions. Chaotic systems are mathematically deterministic-that is, they follow precise laws, but their irregular behavior can appear to be random to the casual observer.[13]

There are a variety of dynamic processes that can progress into chaotic behaviour and that can be used to develop a musical material of such nature. Examples of this are: Period Bifurcation, Iteration and Strange Attractors, amongst others. In this particular research, the method that will be employed to generate a material with chaotic characteristics is iteration.

Iteration, as defined by J. Briggs and F.D. Peat, is “a feedback that implies the continuous reinsertion of what happened before”.[14] Such phenomenon can be found almost anywhere such as in: meteorological events, cyclical replacement of human-body cells, population growth and language. I will now expose how this process was used in order to generate a musical material of such characteristics.

In Search of an Iterative Process that Tends to Chaotic Behaviour Within the Chromatic Scale: The Moebius Progression

To create a musical material with an inherent chaotic behaviour within the twelve-note context is not an easy or, at least, evident task. The reason for this is the

[13] “Glossary”, in National Center for Education Statistics. <http://nces.ed.gov/NCESkids/Glossary.asp#C> [accessed 15th January, 2005]
[14] John Briggs and F. David Peat, *Espejo y Reflejo: Del Caos al Orden.* Gedisa, Barcelona, 1994. p. 66 (translation is mine).

regularity of such a set: the set is modular (i.e. it repeats itself over and over again) and its module – which is of course base twelve – is a regular one (it can be symmetrically divided into two, three, four and six). Hence, clearly repetitive patterns, which become regular self-evident repetitions, can easily appear and destroy an iterative process with a more complex behaviour. A clear example of the system's tendency to stability is when, during the European medieval period, an effort to emancipate music from the tritone *(diabolus in musica)* was made; it was impossible to achieve such task.

After a period of empirical experimentation, I came up with a ten-tone row[15] that satisfied my intention of creating a non-evidently regular note progression. The series is:

Or, put in numbers using the nomenclature used by Allen Forte:[16]

1 2 4 8 5 10 9 7 3 6

Before going on with an analysis of the progression, I will explain how the Moebius Progression was constructed. As previously mentioned, the idea was to construct the tone row by means of iteration. Hence, what I did was to decide what kind of function I would iterate. I chose the simplest operation: adding. To do so, I substituted the notes for Allen Forte's numerical classification (C=0, C#=1, D=2, etc.). Since Forte's nomenclature for the note C is zero, and adding nothing is arithmetically trivial, I took away the note C from the system. It is important to note that, by doing this, I am provisionally converting my modular system from base 12 (the chromatic scale) into base 11 (without C). This will have implications that will be discussed when I analyse the properties of my note progression. For now, I will only say that this step allows the note progression to be developed.

The simple function I decided to iterate is:

[15] I will explain why I call it "Moebius Progression" when I perform the structural analysis.

[16] *Cf.* Allen Forte, *The Structure of Atonal Music.* Yale University Press, New Haven, 1973.

$2x=y$

x is the initial figure of the progression and y its next value. Then, for the following iteration, I substituted x with the value of y obtained in the previous operation:

$y=x_1$
$2x_1=y_1$

I continued in the same manner, substituting the latest value of y to the new x in the next operation. I started my progression with note 1 (C#) and, then proceeded to carry on with the iteration of the mentioned function. The result is as follows:

$x=1$
∴ Tone row: 1

$2(1)=2$
∴ Tone row: 1,2

$2(2)=4$
∴Tone row: 1,2,4

$2(4)=8$
∴Tone row: 1,2,4,8

$2(8)=16$
∴Tone row: 1,2,4,8,5

Note that in the last step of the iteration, the new result for "y" (16) is not equal to the new number of the progression (5). The reason for this is that we are not dealing with a linear system but with a modular "clock-like" one. In a modular system, when a number of adding (or subtracting) steps exceeds the capacity of the module, the count begins again; thus, in module 11: 12=1, 13=2, 14=3, and so on and so forth (note that, if I had a mode 12 system, not having eliminated note C, the note values would have been: 12=0, 13=1, etc.). Bearing this in mind, the continuation of the iteration process is:

2(5)=10
∴Tone row: 1,2,4,8,5,10

2(10)=20
∴Tone row: 1,2,4,8,5,10,9

2(9)=18
∴Tone row: 1,2,4,8,5,10,9,7

2(7)=14
∴Tone row: 1,2,4,8,5,10,9,7,3

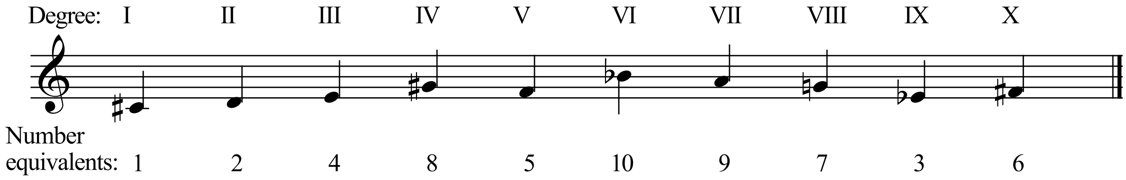
2(3)=6
∴Tone row: 1,2,4,8,5,10,9,7,3,6

2(6)=12
∴Tone row: 1,2,4,8,5,10,9,7,3,6,1

Since the last result is equal to the initial value, if the iteration was carried on, then the series of numbers would repeat itself. Hence, the tone row was completed when number six appeared and, the last number, one, can be discarded. This leaves us with the series previously exposed.

Structural Analysis of the Moebius Progression

For clarity's sake, I will begin with re-exposing the relevant note progression with some additional information: the degree that corresponds to each pitch (in capital roman numerals) and the note's number equivalence in Forte's categorization:



As can be seen in the above figure, the pattern between the first four degrees of the progression is quite evident: the second degree is twice the number of the first one, the third degree twice the second and the fourth twice the third. Such pattern is broken on the fifth degree and recovered between the fifth and the sixth one. However, the later pattern is not completely followed since there is a property between degrees II-IV that is not recovered: the fact that the interval between third and fourth degrees is twice as big as the interval between second and third degrees which, in turn, is twice the size as the span between first and second degrees (i.e. two tones, one tone and a half-tone).

Surprisingly enough, in spite of the linear construction of the progression, the second half of the tone row (degrees VI-X) is inversely symmetrical, as it has exactly the same structure of the first half, but in a descending direction.

However, this symmetry is not limited, as in symmetric serial twelve-tone rows, to the first half of the progression *versus* the second half of it. It is here where another consequence of having used a modular system of base 11 (instead of the chromatic one which is base 12) appears. The symmetry actually occurs between degrees I-VI and its inverse between degrees VI-I:

Degree	I	II	III	IV	V	VI		VI	VII	VIII	IX	X	I
Note Name	C#	D	E	G#	F	Bb		Bb	A	G	Eb	F#	C#
Note Number	1	2	4	8	5	10		10	9	7	3	6	1
Difference	**+1**	**+2**	**+4**	**-3**	**-5**			**-1**	**-2**	**-4**	**+3**	**+5**	

From this we can derive that the structural properties of the relevant series can be reduced to the first six degrees of it and that the rest of the sequence is merely a transformation (inversion) of this segment starting at the sixth pitch and ending on the first pitch. Thus, if we iterated our generating function infinitely, the result would be a never-ending loop that passes through both the minimum expression of the tone row (degrees I-VI), and through the inverse of such minimum expression (degrees VI-I). Hence, the progression's name alludes to a Moebius Strip.[17] It becomes relevant to understand why such an inversion, as opposed to symmetrical twelve-tone rows used in serialism, has a common note (Bb).

The fact that the symmetrical segments of the pitch progression are both six notes long immediately remits us back to the dodecaphonic tone rows and, to the fact that the progression was constructed outside the dodecaphonic system. However, although the iteration performed over Forte's nomenclature was valid within a base eleven modular system, this would not have been the case if the dodecaphonic set were taken into account. This is due to the fact that a gap was created by taking away note C from the system: the set was no longer chromatic.

It now becomes relevant to justify why, given that the note progression is constructed within a modular system base 11, it is being analysed under base 12 criteria. If the ten-tone row were to be used as the Universe[18] for composition, then it would make sense to analyse the system within the context of a module base 11. In such a case, pitches B and C would not exist and, hence, transposing the progression would be impossible (like for instance within modal compositions that only allow internal modulations[19]). However, as it will be later exposed and discussed, the iteration process was carried on further in order to develop transpositions and harmony that are congruent with the construction of the exposed note progression and with its structural nature. It is with the first step of the extension of the musical material, the transposition (to any pitch other than C#), where at least one of the two missing pitches (and in most cases both) is reinserted into the picture. Thus, the system is not only base 11 as when the tone row was constructed, but it is a module 11 set spinning inside a base 12 Universe.

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[17] "The Moebius strip or Moebius band is a surface with only one side and only one boundary component. It has the mathematical property of being non-orientable. It was co-discovered independently by the German mathematicians August Ferdinand Möbius and Johann Benedict Listing in 1858. A model can easily be created by taking a paper strip and giving it a half-twist, and then merging the ends of the strip together to form a single strip..." . "Moebius Strip" in *Wikipedia*. <http://en.wikipedia.org/wiki/Moebius_strip> [accessed February 17th, 2005]

[18] I am using the term within the context of Set Theory.

[19] For instance a tune which is mainly in f# Phrygian and modulates to G Lydian and E Dorian; modes that occur within the context of the D major scale.

Having explained the reason why the material has been analysed within the chromatic context and, within such a context, having exposed its structural characteristics, it becomes relevant to reconsider the objective of the system's construction and ask: has such a regular material any chaotic behaviour?

Where Is the Chaos?

At first glance, our present system seems to be an extremely simple one. This contrasts with the intuition concerning the degree of complexity that a dynamic system must have in order for it to be deterministic and yet behave in an unpredictable manner. In fact, a chaotic system need not be complex at all. An example of an extremely simple chaotic system can be seen in an experiment carried out by four researchers at the University of California, Santa Cruz. These researchers graphed the time intervals between falling water drops from a dripping tap. Surprisingly enough, after a while, the graph was extremely similar to a cross-section of a strange attractor known as Henon's Attractor. When the initial conditions (i.e. the pressure of the water) where slightly changed, cross-sections of other (previously undocumented) strange attractors appeared.[20] Thus, the system cannot be discarded as non-chaotic by arguing its simplicity.

Another objection that might be raised towards the lack of chaotic behaviour within the relevant note series is the fact that the progression, with the exception of the inconsistence of pattern previously pointed out (which is soon enough 'domesticated' by the apparition of an inverse second half of the tone row) has a very clear and organized behaviour. Nevertheless, this is due to the fact that the iteration started at a point where the system behaves stably. As will be seen, this is due to the initial conditions (it is important to bear in mind that chaotic systems are sensitive to such circumstances) and not to the system's inherent chaotic behaviour.

The first thing that must be done is to identify which are the initial conditions. We can discard as an initial condition the function to iterate, as it describes how the system behaves and, changing it would change the system itself. With the same logic, we can discard the fact that we are using a modular system base 11 inside a base 12 one

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[20] John Briggs and F. David Peat, *op. cit.*, p.88.

because this also changes the nature of the system. This leaves us with two parameters to manipulate: the initial value from where to start the iteration and the note we took away in order to make our system base 11.

Let us start by examining what happens if we start the iteration by the other values of our system. The results are as follows:

a)	**2**	**4**	**8**	**5**	**10**	**9**	**7**	**3**	**6**	**1**
b)	**3**	**6**	**1**	**2**	**4**	**8**	**5**	**10**	**9**	**7**
c)	**4**	**8**	**5**	**10**	**9**	**7**	**3**	**6**	**1**	**2**
d)	**5**	**10**	**9**	**7**	**3**	**6**	**1**	**2**	**4**	**8**
e)	**6**	**1**	**2**	**4**	**8**	**5**	**10**	**9**	**7**	**3**
f)	**7**	**3**	**6**	**1**	**2**	**4**	**8**	**5**	**10**	**9**
g)	**8**	**5**	**10**	**9**	**7**	**3**	**6**	**1**	**2**	**4**
h)	**9**	**7**	**3**	**6**	**1**	**2**	**4**	**8**	**5**	**10**
i)	**10**	**9**	**7**	**3**	**6**	**1**	**2**	**4**	**8**	**5**
j)	**11**									

As can be seen, points a) to i) are quite similar to the original tone row. They all have the same kind of inverse symmetry and, although in a different order, exactly the same intervals. Thus, the structure of the set is preserved and the different progressions are mere rotations. Until then, nothing chaotic can be seen in the system's behaviour.

However, the iteration performed with eleven as our initial value throws out a very different result: point j). The system falls into a catastrophic fold: not only does the structure disappear, but also we do not go further than the initial value. Our first chaotic behaviour appears. The explanation for this is very simple: by altering the chromatic set, by excluding number zero from our set, the representation for the unison,[21] adding

[21] Note that, since we are working with pitches without a specific placement in the register, an octave and the unison are exactly the same thing.

zero, was taken out. However, such a concept cannot be eradicated from a modular system and this property migrated into number eleven by means of going around the system and returning to the same point.

Having explored the possibility of changing the initial value of the iteration, let us explore what would happen if the gap in the chromatic scale was left out by taking away another note instead of C. It remains obvious that the structure of our (base eleven) system does not change by the choosing of a note to leave out. However, since we are using a numerical representation of our notes and, by doing so, giving a specific hierarchy to each note (the amount each one adds), the initial conditions are in fact changed by the choosing of a note to leave out. Moreover, by keeping our module base eleven and keeping zero (C) inside the system, we are increasing the probability for the system to fall into a catastrophic fold. This, due to the fact that both representations of the unison, zero and eleven, are threatening to make the developing of the series collapse and, for the first time, we are going to have a gap in our scalar-numerical representation of the chromatic scale (Forte's nomenclature).

Since we are adding in order to get each value and, since now our system is lacking a number that could appear as a result of a previous iteration, it is important to establish a new law as to how the system would behave in this case. What I decided to do is to represent the adding function as "jumping *x* number of steps" inside my modular system.

Bearing this in mind, let us see what happens by eliminating different notes and shifting our starting point. If we iterate our function ten times with these initial conditions: Starting Note= 1 and Missing Note=2, then we get the following result:

1 3 6 0 0 0 0 0 0 0

As can be seen, the iteration throws out quite a different result from the Moebius Progression. Not only the contour of the first four notes change drastically, but after the fourth pitch our iteration loops around a sole pitch: C. Obviously, if we carried on the iteration, the same pitch (note zero) would appear over and over again. This is quite contrasting with what would happen if we carried on iterating the initial conditions that gave birth to the Moebius Progression: it would repeat itself, from beginning to end, over and over again.

In order to make both cases comparable (they obviously are since they both come from the same process) we must distinguish two stages of our iteration. In the first one, the structure resulting from the initial conditions is defined. In the Moebius Progression and in our previous example, the resulting structures would be:

1) 1 2 4 8 5 10 9 7 3 6

2) 1 3 6 0

In the second stage, the system loops around either part of the resulting structure, or the whole structure. In our examples, the latter stage would be:

1) 1 2 4 8 5 10 9 7 3 6

2) 0

In the previous examples, only two cases of the system's possible behaviour were treated: where the looping section is equal to the original structure and, where the looping section is a single note. Following is an example where the looping section is only a segment of the original structure (Starting Note=4, Missing Note=3):

Whole progression: **4 8 5 10 9 7 2**

Looping section: **5 10 9 7 2**

Having defined both stages of the iteration process (definition of the original structure and looping section), let us look at the results thrown out when the starting pitch is kept as low as possible, but different to zero, and the missing pitch is varied:

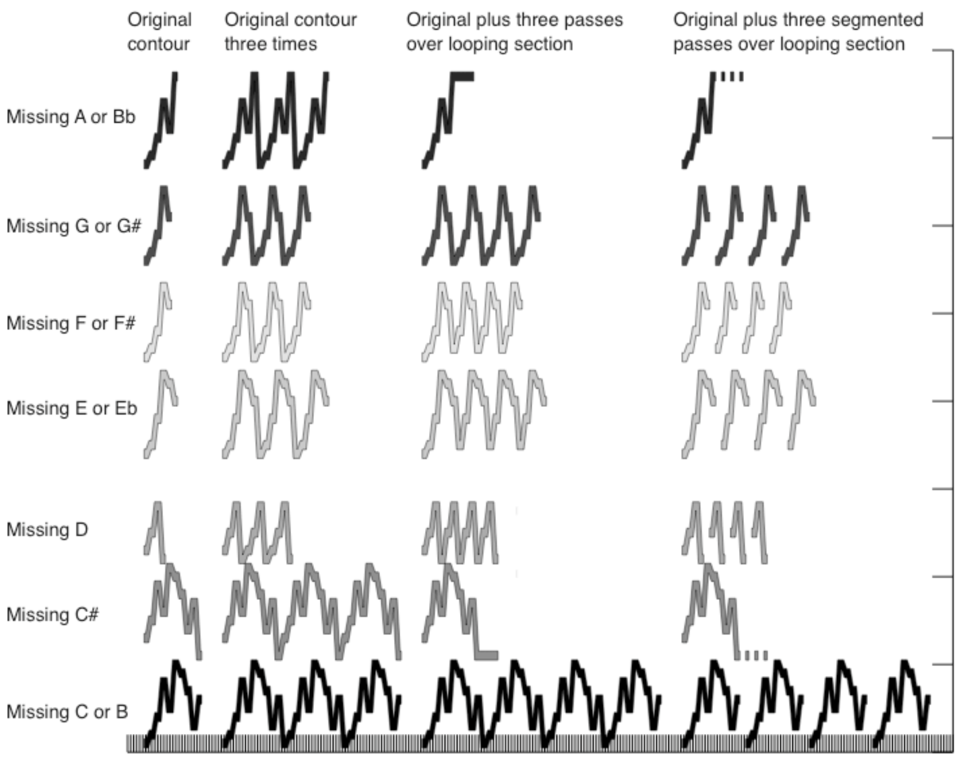
Note Taken Away	Resulting Progression										Progression's Loop Section				
C (0)*	1	2	4	8	5	10	9	7	3	6	Whole progression				
C# (1)	2	4	8	5	10	9	7	3	6	0	0				
D (2)	1	3	6	0							0				
Eb (3)	1	2	5	10	9	7					2	5	10	9	7
E (4)	1	2	5	10	9	7					2	5	10	9	7
F (5)	1	2	4	9	7						2	4	9	7	
F# (6)	1	2	4	9	7						2	4	9	7	
G (7)	1	2	4	9	6						Whole progression				
G# (8)	1	2	4	9	6						Whole progression				
A (9)	1	2	4	8	5	11					11				
Bb (10)	1	2	4	8	5	11					11				
B (11)	1	2	4	8	5	10	9	7	3	6	Whole progression				

Keeping in mind that, together with altering which note we omit, we can also change the starting point of the iteration (which would throw permutations of the above results plus new progressions) we can already see in the former results how the system's potential for chaotic behaviour is coming through. Let us comment on these results before going on with altering both initial conditions.

As can be seen from the above table, outside of the original tone row, only one progression (the last one) preserves the same structure as the original one. Outside these two cases, the original series' property of containing its inverse is lost. Furthermore, the length of the progressions begins to vary as well as the inner interval structure of the new progressions (the latter is most notable in the progression constructed without note D). However, perhaps the most drastic effect of altering our gap in the chromatic set takes place in the nature of how the progression would loop if we continued our iteration of the generating function indefinitely after the progression started repeating itself (bear in mind that our original Moebius Vortex would have repeated itself exactly over and over).

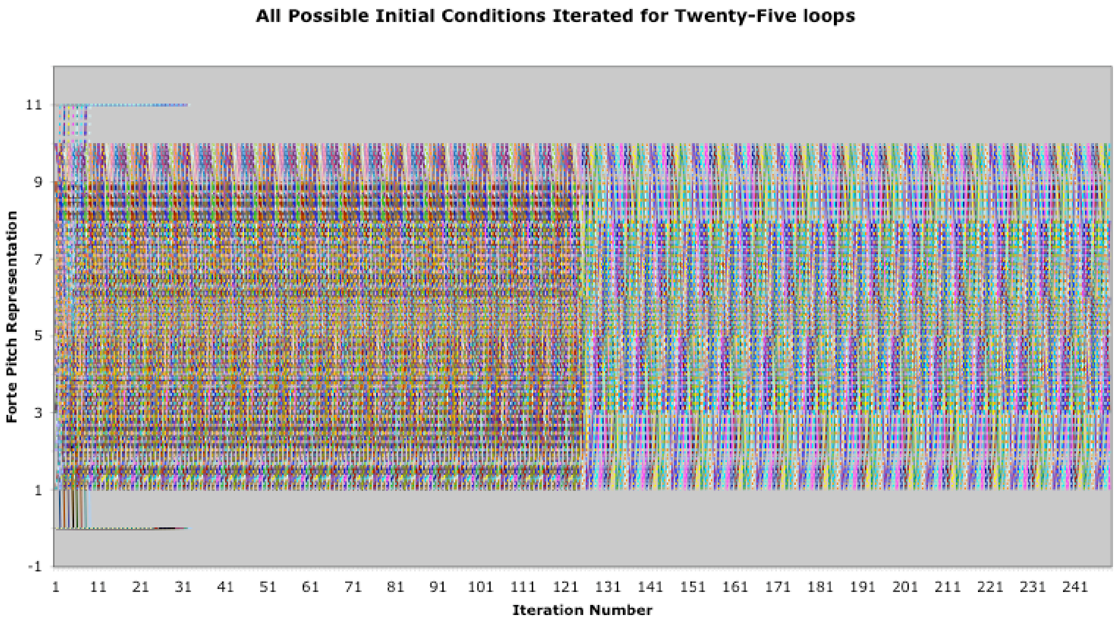
This graph describes such effect quite clearly:

* Original



As can be seen, the first column of differently shaded patterns shows how the altering of initial conditions affects the contour line of each progression. We encounter seven different cases with different degrees of similitude. The second column describes what the contour would look like if the looping section of the tone rows were equal to the whole progression (this is the case when notes C, G, G#, and B are removed). Although this column is not relevant to how the system functions, I find it pertinent to place such contours as a point of comparison to the next column. As can be seen, on the next column the progressions are displayed once completely and followed by three loops of their respective recurring section. In this case, we can see how a drastically new behaviour appears: when notes C#, A or B appear, the looping section stays in one note causing the progression to collapse. The final divergent behaviour, the most common, appears when the missing note is either D, Eb, E, F or F#. In these cases, the contour is preserved almost as in the original contour, but it looses a pitch when the progression goes into its looping section.

We have explored how each of the initial conditions (starting point or missing note) can affect the system. It now becomes relevant to explore the system's overall behaviour. To do so by analysing a chart of all possible initial conditions and their respective iterations would be quite unpractical since there are a total of 132 possible initial conditions (for an account of these, see Appendix A). A clearer idea of the possibilities of the system's behaviour can be expressed as a graph. The construction of such image was done following these steps: 1) first, all possible initial conditions were iterated until they completed twenty-five periods;[22] 2) each individual progression was assigned a different kind of line that would represent it and; 3) all the progressions were plotted on a Cartesian plane of which the x-axis represents the different pitches (in Forte's nomenclature) and the y-axis stands for the number of iterations each sequence has followed. Here is the resulting graph:



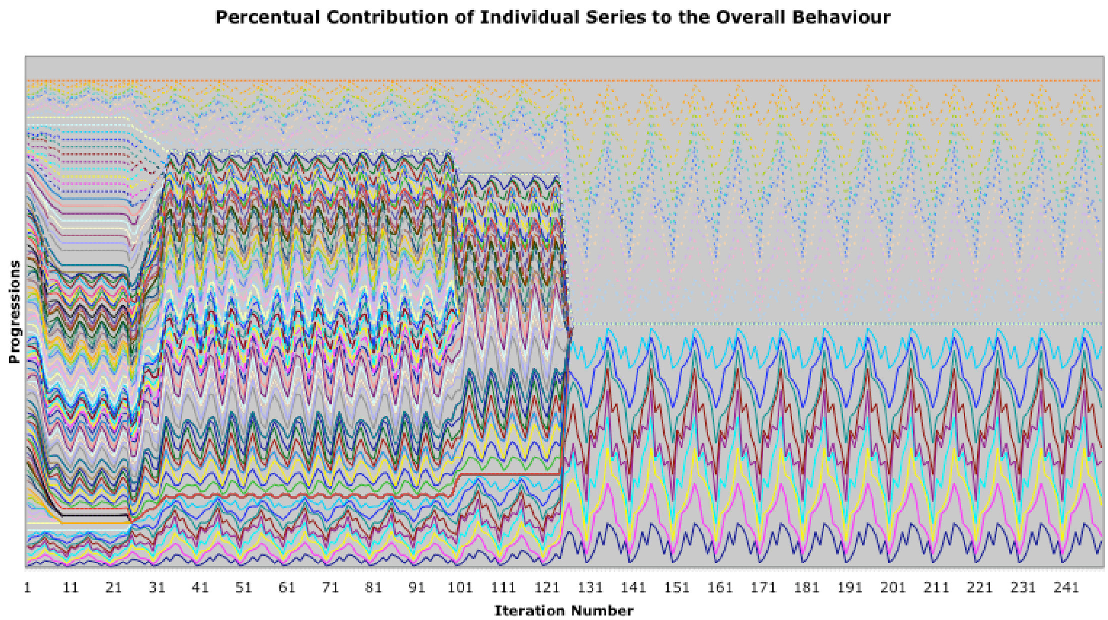
This graphic representation of where the system might be found at any iteration has still too much information to give us a clearer idea of the system's possible behaviours. However, we can already roughly distinguish four quite different stages: from iterations 1-10, 11-30, 31-125 and 126-250, approximately. These phases can be easily differentiated in the original data as sections with a different amount of progressions (since the lengths of the progressions vary, some take much longer to reach 25 loops). In addition to this, we can also say that the transition sections between these four blocks have, each, a unique behaviour. Another thing that can be already observed in the graph is that the symmetry present in our Moebius Progression, which

[22] The first period is the complete progression and the subsequent ones are only the looping section of the corresponding progression (as seen before, the looping section can be the whole progression or a segment of it).

was lost in most of the individual progressions, still seems to be a property of the overall system's behaviour.

To further understand these results we encounter a problem: since we are trying to understand how the system might behave –as opposed to how it behaves under certain conditions– reducing the amount of information does not serve our purpose. In other words, if we chose a quantitative approach in order to analyse our system, we may get an accurate and quite simple result for any particular case of our iteration. However, to further understand the system's behaviour, as a whole, we need to carry out our analysis from a qualitative point of view.

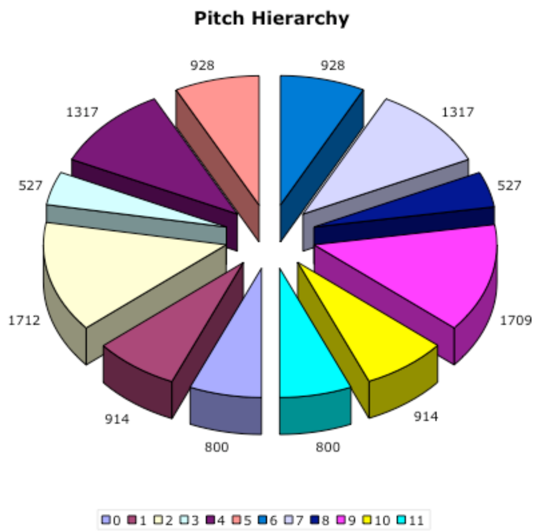
The first thing that frustrated me while trying to figure out the meaning of the previous graph was the impossibility to find out whether the system has any internal movement after the first ten iterations (where on each iteration one progression reaches pitch C and another one pitch B), or whether the adding up of all progressions gives as a result a constant cluster which only varies with respect to its density per pitch. To answer my question, I first tried to generate a graph that gave me an overlook of the whole system while still differentiating the individual series. How to achieve this was not at all evident. Since I already had my data inside a spreadsheet-software that has several graphing capabilities, I decided to browse the software's help to find out if there was such an option. I came across a graph that showed the percent contribution of each series' tendency to the overall behaviour. Here is the resulting image:



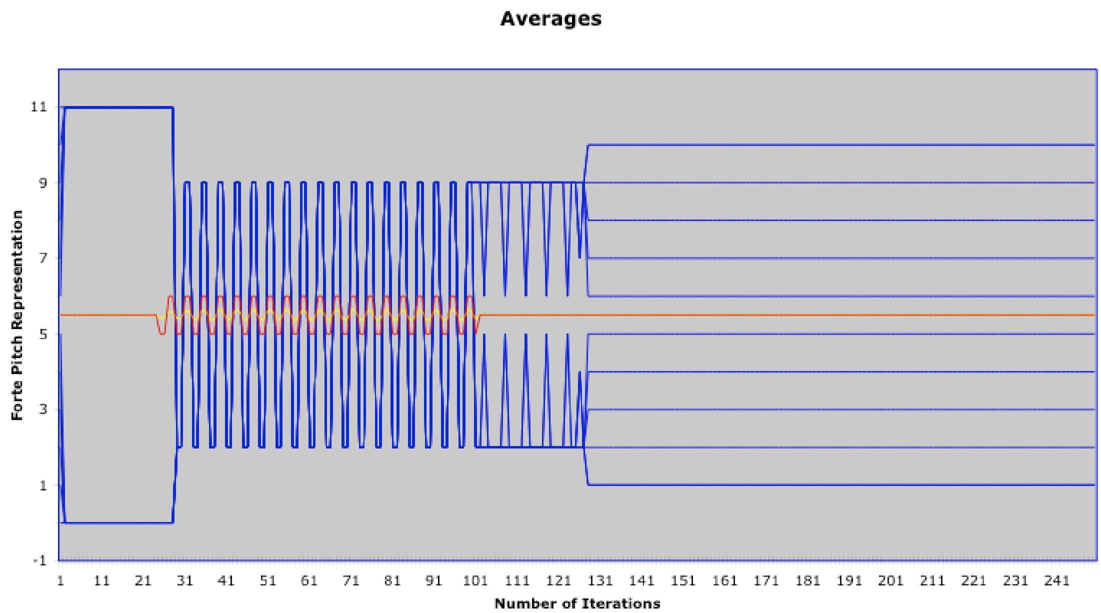
Before commenting the graph, it is relevant to point out two things: 1) the dotted lines do not have any special meaning; the reason why they appear is that I wanted a different kind of colour for each progression and, since there are so many progressions, differentiation by colour was not sufficient; and 2) the progressions are a chained repetition of a basic pattern whose peaks and valleys become more exaggerated after each transitional section; however, if, after a transitional section, a progression's basic pattern changes or is covered by another progression, then it means that the progression has finished its 25 loops and has ceased to exist.

This said, the graph shows very clearly the decrease in density of the system as the progressions carry on. It also shows how the system has stages where a very stable behaviour can be observed but each stable stage can now be observed as having a unique quality. Concerning the unstable stages, that is, the transition between stable stages, it can be observed that they occur during a few iterations and, therefore, have a very drastic effect on the system. Another quality that can be observed is how longer progressions, the ones that take more iterations to complete their 25 loops, have very little influence on the system at the beginning of the process and, as the system progresses, they gain more and more influence. This last asseveration is backed up by the fact that the pitch sequences' peaks and valleys become more exaggerated as the system moves into more advanced stages which means that their percent influence on the system is greater.

Though not conclusively, this last characteristic points out to the possibility of having a hierarchical behaviour within the system (since not all sequences have the same pitches). It is obvious that, if we take into account the dodecaphonic system this, is the case since, after iteration 34, pitches 0 and 11 disappear. To answer my question with respect to the rest of the pitches, I decided to calculate the occurring of each pitch throughout my iterations. The result is as follows:



From these results we can see that the system does have a hierarchical behaviour. Furthermore, since ten of the chromatic pitches occur throughout our iterations (C and B disappear quite quickly), there must also be a considerable amount of internal movement between the pitch interrelations. To corroborate this, it should be enough to extract the basic statistical data out of our system (namely mean, median and mode). Let us examine such information as the iterations occur:



The first thing we observe is that, as expected, the system's basic statistical behaviour supports our previous hypothesis on a regular behaviour during different stages of the iteration process and of a differentiation between these stages. More information can be obtained by interpreting the different averages.

The yellow line represents our first average: the mean (such line only appears when it is not equal to the red one that covers it). The mean is only differentiated from the median between iterations 24-101. Outside of these sections, our average is represented by a straight line. If we have a symmetrical system, it makes sense that the average of it is a straight line. Therefore, we can say that the system is not entirely symmetrical. The median will give some light as to what is happening where the symmetry is challenged.

As can be seen, our mean line is very similar (most of the time identical) to our median line (colour red). Now, when they are not identical, their contour behaviour remains analogue. Furthermore, while the mean's fluctuation only suggested that the system's instantaneous (i.e. on any given single iteration number) symmetry was lost, the median's behaviour confirms the reason for this: the amount of values above or below the midpoint becomes unequal. Nonetheless, such inequity shifts between above and below the midpoint with equal frequency. From this we can derive that the system's symmetry, between iterations 24-101, has undergone a phase shift. The fact that a phase shift exists suggests that such particular behaviour has something to do with the different single progressions' lengths. After examining the data spreadsheet from which the graph was constructed,[23] it was observed that such phase shift occurs between when the first progressions (the ones who's looping sections are either zero or eleven) cease to exist and when, 75 iterations later, the next group of progressions disappear. Therefore, we can say that the system behaves differently depending on how many progressions are still completing their 25 loops.

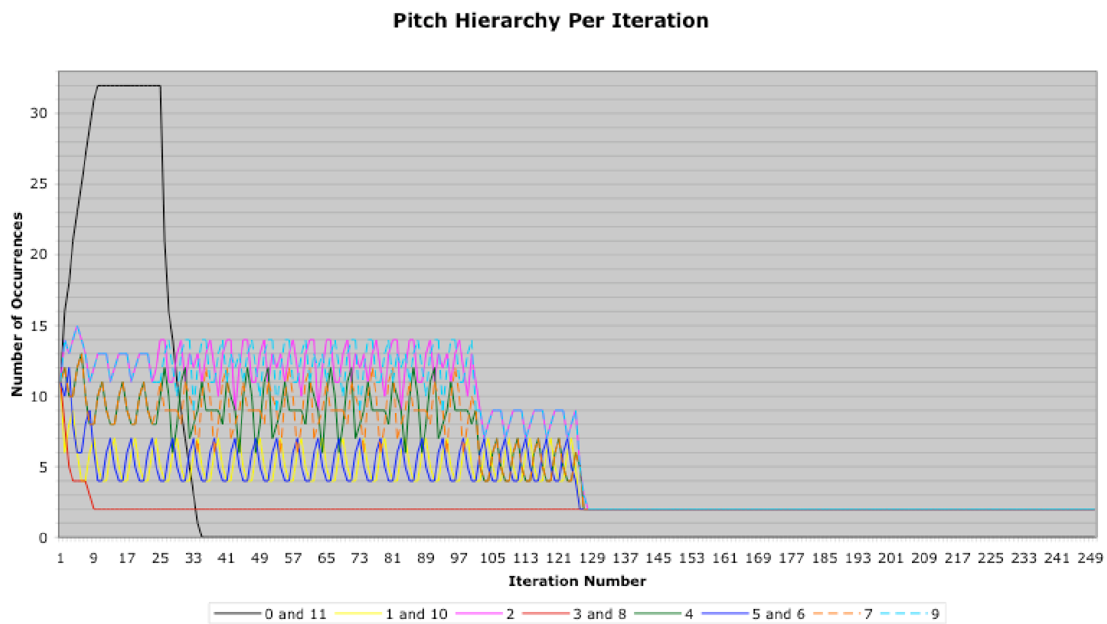
Further evidence of this can be derived from our last average: the mode (represented by blue lines in the previous graph). With the mode, we observe six different behaviours: a non-hierarchical iteration (the first one); a section where the mode falls on pitches zero and eleven (iterations 2-28); a segment where the mode oscillates between pitches two and nine (iterations 29-100); a part where there are two hierarchical constants, pitches two and nine again, and where periodically the mode falls on two more notes, five and six (iterations 101-125); a moment similar to the

[23] The spreadsheet is not included in this essay since it would take 100 (4 of height by 25 of length) A4 pages to display, a format completely incompatible with the present medium.

previous section but, with pitches four and seven, instead of five and six (iterations 126-127); and a final section where the system falls into a constant non-hierarchical behaviour where all the appearing notes (pitches zero and eleven disappeared a long time ago) have the same weight (iterations 128-250). Again, after comparing where the changes in the mode's behaviour occur, we can say that they are linked to the amount of progressions still waiting to complete their 25 loops.

After the previous observations, we can say that the system can behave quite differently depending on the stage it is at. Furthermore, we can say that the system's distinct behaviour is directly linked to the amount of sequences that are looping at that particular moment. Another quality of the system is its stability within each of the sections. Finally we can say that, by far, the mode renders the most information from the system.

This, in combination with the information concerning the pitch hierarchy within the whole system, led me to see that the pitch hierarchies actually have a lot of internal movement. This led me to ask myself what happened with the hierarchy of all the pitches in the different sections. To do so, I computed the hierarchy of each pitch per iteration and plotted it into the following graph (note that some pairs of pitches have identical hierarchy throughout the system and, therefore, are represented by the same line):



Just as in the graphs depicting the statistical data (averages and percent contribution to the overall behaviour), we can distinguish five different behaviours intrinsically linked to the amount of progressions present.

The latter graph can be analysed and further qualitative analysis may be performed on the system. However, the purpose of this chapter was to find out if there is a chaotic potential inherent in the constructed system. I consider we have enough information to do so. Moreover, we have already observed that the extracted data points in the same direction: the system's overall behaviour is very sensitive to the amount of progressions being iterated.

To answer the question that served as a title to this section, we need to know what conditions a system must posses in order to be classified as chaotic. For a system to fall into this category it must: be sensitive to initial conditions, be topologically mixing, and its periodic orbits must be dense.[24] Hence, the constructed system must now be examined in order to find out if it fulfils such characteristics.

The first requirement, sensitivity to initial conditions is perhaps one of the clearest properties of the system. The starting note and missing pitch, the initial conditions, can have a drastic effect on how a chosen progression behaves. Let me illustrate this with an example.

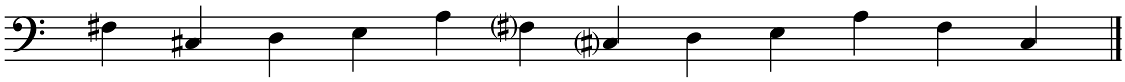
Suppose that, within a compositional plan, a *cantus firmus* is desired and that, as material, one of the sequences from the present system will be employed. Let me choose three pairs of initial notes –starting note and missing pitch, respectively– that will make my point clear: (6-0), (6-4) and (6-8). The resulting pitches after twelve iterations are:

Initial conditions: (6-0)

Initial conditions: (6-4)

[24] "Chaos Theory", in *Wikipedia* <http://en.wikipedia.org/wiki/Chaos_theory> [accessed on January 24th, 2006]

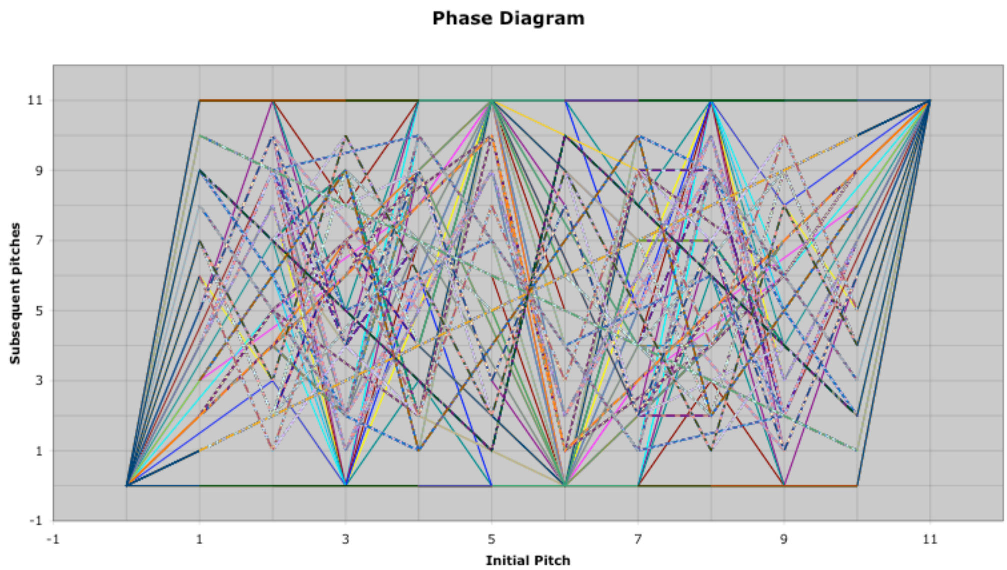
Initial conditions: (6-8)



Of course, provided the necessary craftsmanship, the three results can be used to make music. However, for the hypothetic *cantus firmus*, the second result is unusable. As to the first and third results, at first sight, the music they might suggest is entirely different. Therefore, it can be concluded that the system is sensitive to initial conditions.

Nonetheless, it should be stated that the system could also behave equally provided different initial conditions (e.g. the Moebius Progression can be generated with the following different initial conditions: [1-0] and [1-11]). Furthermore, every resulting progression can be generated by, at least, two different pairs of initial conditions.

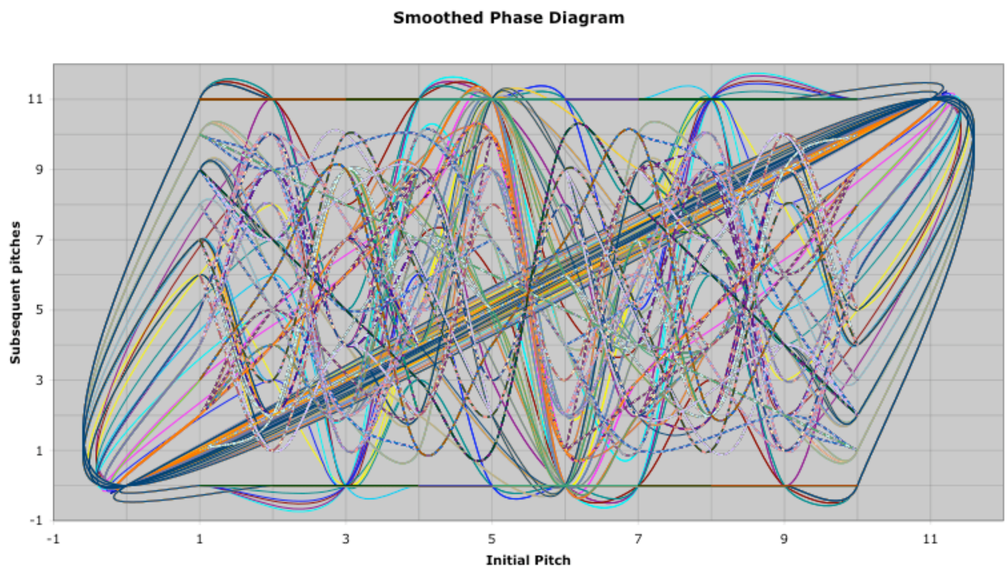
The second, previously mentioned, characteristic of a chaotic system –that it must be topologically mixing–[25] is definitely present in our system. For an account of this, a diagram of the system's phase map is presented:



[25] According to *Wikipedia* (*ibidem.*) a topologically mixing system is one that will evolve over time so that any given region of its phase space will eventually overlap with any other given region.

It must be said that the fact that the system is topologically mixing is not strange at all when working with the chromatic set. Since there are only twelve positions where the system can be found, overlapping is much easier to achieve than to avoid. Nonetheless, the way that such overlapping takes place in the present system resembles the kind of strange attractors, which appear in discrete systems.

The third requisite that must be fulfilled by a system for it to be considered chaotic, as mentioned earlier, is that its periodic orbits must be dense. Since the previous phase diagram was plotted out of 12,393 values that can only fall on 144 places, the system must be quite dense. To clarify such density, a phase diagram was plotted in which the lines representing the pitch progressions were smoothed (note that, although the following graph depicting the smoothed phase diagram resembles a strange attractor in a continuous dynamical system, such similitude is due to the smoothing and not to the system itself):



Since the above lines' curvatures are dependent not only on two immediate values but, also on their adjacent ones, we can obtain an artificial differentiation that shows the presence of much more progressions. Still, the diagram does not reveal the whole density since identical sequences are still fully overlapped.

As conclusion, we can say that the developed system does have a similarity with chaotic behaviour. As previously stated, it does fully accomplish two of the characteristics of chaotic systems: it is topologically mixing and its periodic orbits are dense. The system can also exhibit sensitivity to initial conditions. However, it can also behave identically under different initial conditions. Hence, the potential for chaotic behaviour is limited. Nonetheless, the system can be made to behave orderly or irregularly since the choice of initial conditions is, of course, possible when composing.

Whether the system is chaotic or not, the analogue behaviour it can exhibit has proved to be extremely useful as compositional material. Furthermore, the fact that it can behave in both an extremely organized way or in a chaotic fashion, has been extremely inspiring in my composing. This is the main force that has motivated me to take this research to the stage it is at and, although I have already carried out further experimentation within the presented material, both on an abstract (theoretical) and a concrete (compositional) level, I believe that, being a composer, it is wiser to explore the musical possibilities of the presented system before carrying on with a theoretically driven expansion of it. Nonetheless, at this stage, I do find it relevant to open up a series of possibilities for further developing the system.

Etcetera

Although until now the system has been presented as already fixed and closed, due to an analytical necessity, in practice this is not the case. Continuing the expansion of the present musical material is something that has already taken place within my compositional practice and a possibility that has already theoretically been thought off.

For instance, until now, the system has been presented exclusively as a set of possibilities for horizontal development. This possibilities have, until know, been fixed to the numerical abstraction of pitches. Hence, pitches were treated in an absolute fashion (strictly linked to their numerical representation). However, by simply focusing on what such arithmetical values mean musically (that is, intervallic proportions) a step for further development can be immediately foreseen: a transposition system that obeys the same logic as the construction of the system. Such a step was taken, for instance, in *Mechanism for Two Marimbas* (see chapter IV) where, by iterating the interval relationships of the “Moebius Progression” starting at each note of the latter tone row, a transposition matrix was constructed. While such an action only transposes the interval structure of our system, it can have a drastic effect on how the system behaves if one reinterprets the relations in pitch as relations in rhythm.

Another already carried out extension of the present musical material was, following the same logic with which the original system was constructed, iterating one of the system's possibilities (again the "Moebius Progression") into the vertical domain. This gave as a result ten harmonic[26] progressions that preserve the system's horizontal relations both in each progression's density and in the sequences' number of elements. Furthermore, thanks to pitch class analysis, it was observed that each of the ten harmonic progressions preserved the symmetry previously observed in the "Moebius Progression". Part of such an extension was used as harmonic material for *Inertia and some of its exceptions*.

Two foreseen possibilities that remain unexplored but that seem to have an interesting potential are: to change the function whose iterations construct the system, and to iterate intervals instead of single pitches.

In the first case, iterating different functions, the resulting systems remain an open question. However, since this parameter describes the overall behaviour, saying that the system's attractors would turn out quite different is already a conservative starting point hypothesis.

As to working with intervals as initial conditions (an idea that surged during the process of the present research) it appears that the amount of results would square. For instance, one could, instead of iterating a function with integer numbers like in the present research, work with a function involving complex numbers (of which both real and imaginary components should be whole numbers; so that they are compatible with the chromatic set). The translation from interval to complex number does not present a problem since, within the existent music theory, intervals are already being treated as vectors[27] and complex numbers can also be expressed as such.

As can be seen, this research is a possible starting point for explorations of iterative processes that can generate chaotic behaviour within the chromatic set. Since, for a composer, music theory is a tool for a specific goal: writing music; it makes sense to stop here and employ the present theoretical constructs in practice. It is in the inclusion of both theory and practice that a composer can: 1) have a deep empirical understanding of how a musical system, such as the present one, can behave within

[26] The term harmonic is employed because, although there are no tonal functions within the system, the resulting progressions still observe a hierarchical behaviour between their vertical sonorities.
[27] As does Allen forte, *op cit*.

another chaotic process: the organization of sound (and silence) in time and, 2) approach research as a tool to develop musical material that obeys his/her compositional needs.

IV. *Mechanism for Two Marimbas:* An Etude on Re-Articulation

"How much creativity is there in mixing a horse and a rhinoceros to get a unicorn? And these are our fantastic animals!" My father uses more or less these words to express a view contained within a psychoanalytic approach to the nature of fantasy.[28] Under the same kind of logic, but previous to Freudian times, in *Frankenstein*, Mary Shelly created the most famous monster of romantic literature by simply reconstructing the same species: the human being, but from segments taken from different persons (with all the symbolic weight that such an action would carry). In different ways, both Freud and Shelly are showing that creativity is an elaboration of pre-existing elements.

This same emphasis on creativity as re-articulation prevails in contemporary art. While modernist art centred on originality and formal innovation, postmodernism conceives creativity as an innovatory logic or syntax. Thus, postmodernist art incorporates materials and styles from diverse contexts, which, thanks to their re-articulation, are able to express something different to what they originally did.

It is the purpose of this chapter to show how two important influences on my thought, the music of Donatoni and Chaos Theory, are re-articulated into my artistic production.

The Conceptual Material

When I first came across a fractal, the Mandelbrot Set,[29] its visual beauty did not particularly impress me. I was reading a book on Chaos Theory and my attention was focused on the fact that such an image represented a sole iterative function.[30] My

[28] I don't know if he is, in time, paraphrasing another psychotherapist.
[29] Named after the inventor of the set and of Fractal Geometry: Benoît Mandelbrot.
[30] The iterative function for the Mandelbrot set is: $z_n^2 + z_0 = z_{n+1}$ where z_n is a variable complex number and, z_0 is a constant complex number. The iteration takes place by, on the next step of the iteration, substituting z_n by $z_{n=1}$ and getting as a result z_{n+2}. The function is iterated over and over again. The Mandelbrot Set consists of all the results of the iterations for the different values of z_0 (namely all the points in the complex plane) where the results of the iterations do not escape to infinity. John Briggs and F. David

formation as a mathematician does not exceed high-school level and, therefore, I was amazed by the conjunction of similarity and change that my book's diagrams revealed in different scale zooming of segments of the Mandelbort Set, and by the fact that such relation was all contained within the possibilities of a singular function. My thought was drawn to what I had reflected when reading Friedrich Engel's argumentation on the close relation between the origin of the family and the state:[31] I began seeing a loose, though clear, connection between how people relate with members of their family and how they relate in a wider social structure. Furthermore, I could see how this kind of organization repeated itself in bigger kinds of communities (between an individual and a group, between groups, institutions, etc.). Moreover, I began making analogue hypothetical connections with respect to quite different phenomena.

I believe it is already clear that this stream of thought did not follow a scientifically rigorous path. However, it did nurture a sense of belonging and of being part of an interrelated whole which is, I believe, a central idea behind human spirituality. Being a composer, the most logical reaction to such an experience was to incorporate this kind of logic-thought-intuition into my artistic doing.

My starting point for this was the Internet and a "fractal music" query on a popular search engine. My effort was quickly rewarded and I was soon listening to sounds organized by means of fractal-generating iterations. Yet, what I heard was not at all what I expected: most of the time, the sounds did not remit me to the kind of richness I had seen in fractal images. Nonetheless, I must admit that several examples were quite interesting.

Still, music that, in most cases, was not composed after the concept of fractals sounded to me as more closely related to them. A few examples of this are: Ligeti's *Autmone à Varsovie*, Xenakis's *Eonta,* and Donatoni's *Françoise Variationen.*[32]

The next effort in my artistic quest was twofold. On the one hand, I started experimenting with iteration as a means of generating musical material (see chapter III). On the other hand, I analysed (or, in most cases, re-analysed) pieces that I thought were as close as possible to what I was looking for. My analytical criteria focused on the

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Peat, *op. cit.* p. 97where the results of the iterations do not escape to infinity. John Briggs and F. David Peat, *op. cit.* p. 97

[31] Friedrich Engels, *El origen de la familia, la propiedad privada y el estado.* Fondo de Cultura Económica, Mexico City.

[32] For an analysis of three of the *Françoise Variationen* and a relationship of these with fractals, see chapter II.

workings of chaos-order and similarity-change. Finally, I incorporated my interpretations into the application of the developed musical material. This process was first carried out in *Mechanism for Two Marimbas (McAfferty's Lullaby)* (see Appendix B). Following, is an account of how the piece was conceived.

From Concept to Craftsmanship

After exposing the conceptual material behind *Mechanism for Two Marimbas*, a disclaimer is needed. It is not my intention to transpose extra-musical motivations into my compositions, a possibility that remains questionable. While not long ago I thought that my compositional work should follow a mimetic path, my efforts and reflections on this possibility put me on a different position: I now believe that a more ethical approach to writing music (at least abstract music) is to deal with a core principle of it: the unity of content and form as one and the same thing. Hence, I consider my concept as a departing point (inspiration if so wished) that allows me to articulate formal elements according to a particular point of view. Once such an articulation is present, I try to remain congruent to its nature. I say I try to do so because of two conflicting elements: 1) the inexistence of absolute congruence in human nature and, 2) the conscience of the score as a means to obtain an aurally perceived result which, may sometimes be better suggested by slightly deviating from formal integrity (account of such a practice can be found, for instance, in Greek classical architecture, where a harmonious optical perception was obtained through altered proportions at the construction level). This said, I will go on with an explanation (subject to the limitations implied by the last sentences) of how the piece was developed.

The pitch-rhythm material I used for this piece was derived from the Moebius Progression (see previous chapter). However, I did not employ the initial tone row. Instead, I took advantage of the progression's property of conserving the same kind of inverse-symmetry after any kind of rotation, and of the fact that such rotations can be obtained by altering the initial conditions to be iterated. In this way, I used the rotation that gives as a result the inverse interval structure of the original Moebius Progression but starting with pitch Bb. Then, I transposed it a semi-tone higher.[33] Finally I constructed a transposition matrix that served as material for the piece:

Moebius Inverse Matrix in "B"

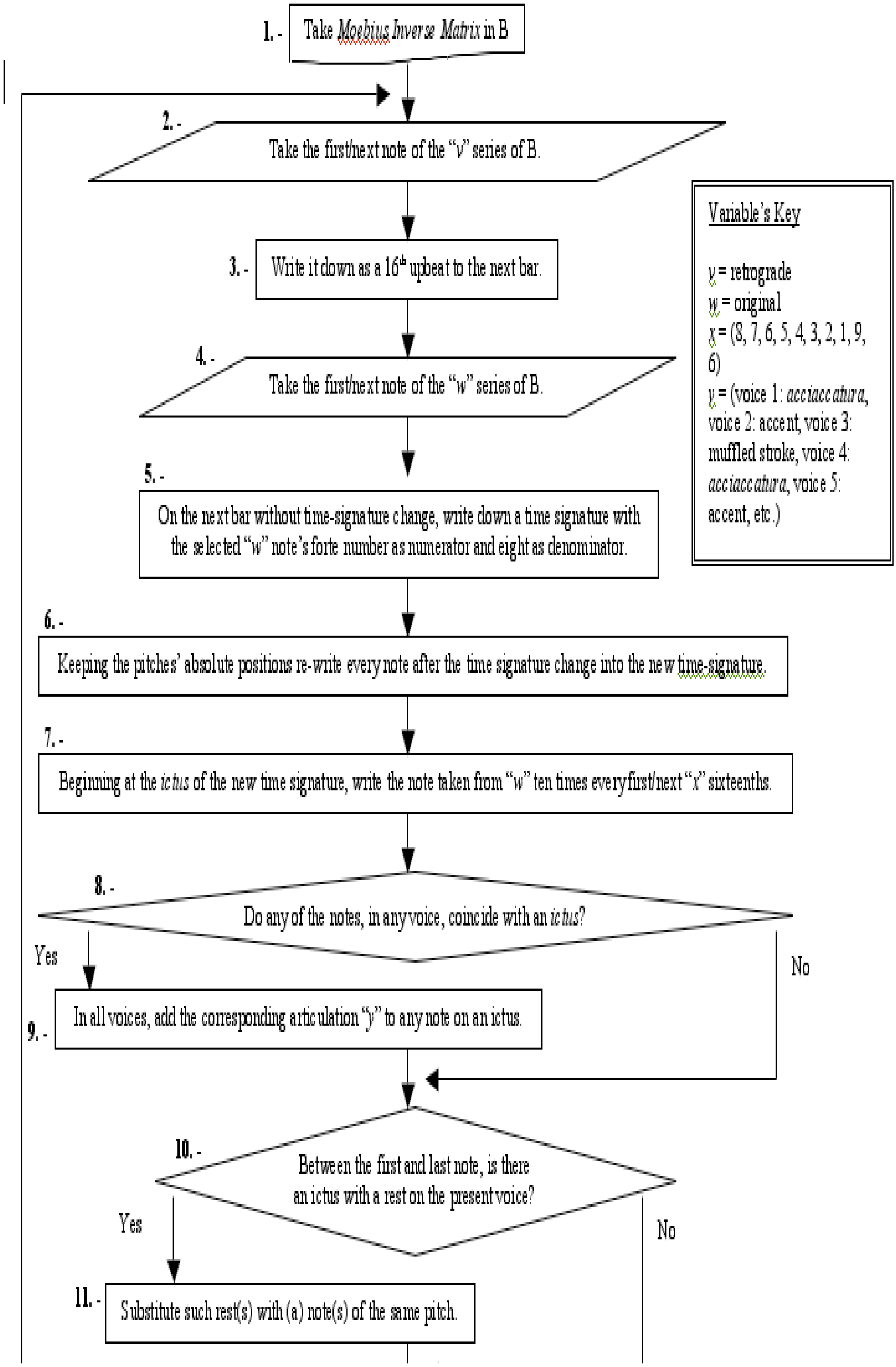
Original - > < - Retrograde

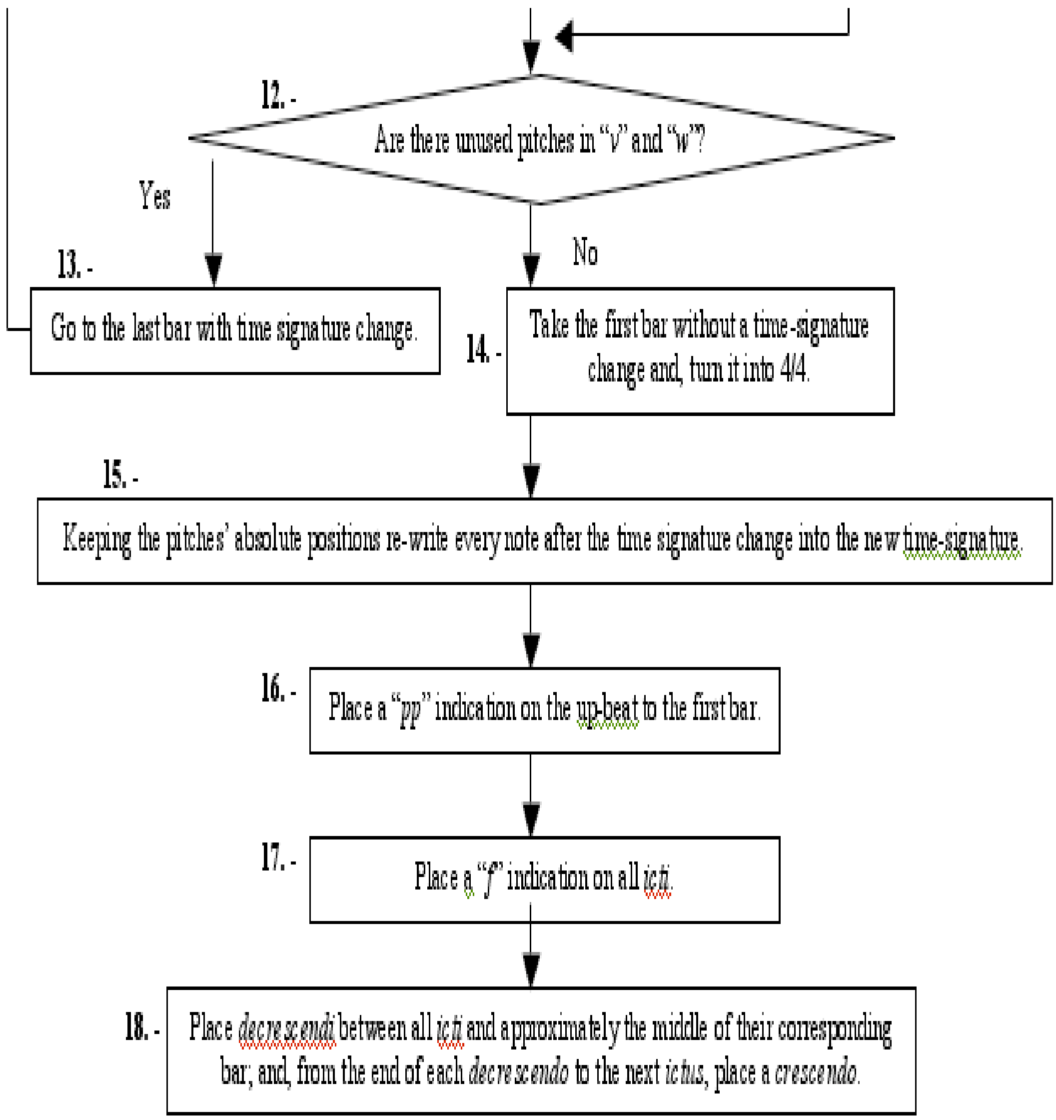
Inverse - > < - Inverse Retrograde

B	Bb	G#	E	G	***D***	Eb	F	A	F#
C	**B**	A	F	G#	Eb	***E***	F#	Bb	G
D	C#	**B**	G	Bb	F	F#	***G#***	C	A
F#	F	Eb	**B**	D	A	Bb	C	***E***	C#
Eb	D	C	G#	**B**	F#	G	A	C#	***Bb***
G#	G	F	C#	E	**B**	C	D	F#	Eb
G	***F#***	E	C	Eb	Bb	**B**	C#	F	D
F	E	***D***	Bb	C#	G#	A	**B**	Eb	C
C#	C	Bb	***F#***	A	E	F	G	**B**	G#
E	Eb	C#	A	***C***	G	G#	Bb	D	**B**

Then, based on the idea of Koch's Curve[34] I constructed an algorithm from which the first section of the piece was derived. I now present the flowchart followed by an explanation of why each step was taken (the numbers on the left hand side of the flowchart correspond to the numbers in the explanation below):

[33] Such decisions were motivated by: a macro plan for a cycle of which the present piece is the first movement and, for a rhythmic reason. The former will not be discussed but the latter will be explained later.
[34] On Koch's Curve, see chapter II.


1. - Take *Moebius Inverse Matrix* in B
2. - Take the first/next note of the "*v*" series of B.
3. - Write it down as a 16th upbeat to the next bar.
4. - Take the first/next note of the "*w*" series of B.
5. - On the next bar without time-signature change, write down a time signature with the selected "*w*" note's forte number as numerator and eight as denominator.
6. - Keeping the pitches' absolute positions re-write every note after the time signature change into the new time-signature.
7. - Beginning at the *ictus* of the new time signature, write the note taken from "*w*" ten times every first/next "*x*" sixteenths.
8. - Do any of the notes, in any voice, coincide with an *ictus*?
Yes
No
9. - In all voices, add the corresponding articulation "*y*" to any note on an ictus.
10. - Between the first and last note, is there an ictus with a rest on the present voice?
Yes
No
11. - Substitute such rest(s) with (a) note(s) of the same pitch.
Variable's Key
v = retrograde
w = original
x = (8, 7, 6, 5, 4, 3, 2, 1, 9, 6)
y = (voice 1: *acciaccatura*, voice 2: accent, voice 3: muffled stroke, voice 4: *acciaccatura*, voice 5: accent, etc.)



1. The first instruction simply asks to use the transposition matrix as pre-existing material.
2. This instruction, as well as the next one, was added after the first version of the algorithm was configured. I felt the necessity to put an extra emphasis on the time-signature structure and, through this instruction, used the retrograde series of B as pitch material for doing so.[35] On each recursion of the algorithm, the corresponding pitch from the relevant progression is selected.

[35] Note the difference between "the retrograde series of B" and B retrograde. The first one makes reference to the series-starting-in-B's retrograde and, the second one to the retrograde starting in B.

3. On each loop's recursion, the time-signature structure will be emphasized by adding an upbeat of a semiquaver towards the next bar. All the upbeats together will complete the retrograde series of B.[36]
4. This instruction refers to the main series that will be used as material for constructing the section. Just as on instruction 2, one pitch per recursion is read.
5. This instruction defines, recursion after recursion, the section's metric structure. Such a structure will be equal to: the series of Forte's numeric representations of the main series (see previous step). In this way, my irregularity with respect to the metric ratio is derived from the main structural material.
6. Since this instruction only has an effect on the recursions that follow the first one, I will explain it after instruction 13 (where the algorithm is sent into a recursion).
7. This step builds a different pedal on (iteration after iteration) all of the notes corresponding to the original series. The proportion of each pedal is isorhythmic. However, the scale changes depending on the recursion number. Such scale is organized so that, after each algorithm's loop, the pedal's frequencies are higher and, on the last two loops, slower again. This was decided in order to obtain a gradual increase of movement and, towards the end of the section, a less gradual decrease of it.
8. This binary decision exists in order to put an extra emphasis on the possible coincidence of the existing isorhythms and the current time signature. In this way, a net-like structure, derived from the interrelation of the independent isorhythms, is constructed.
9. If the previous decision turned out positive. The current provisional time signature is emphasized by adding either: *acciaccature,* muffled strokes, or accents, depending on each voice's corresponding articulation element.
10. This binary decision alters the isorhythmic structure of each pedal. The intention was to keep information of the time signature where the voice was written (the rhythmic ratio will only be explicit the first time it appears) regardless of the fact that the next iterations will change the time signature.

[36] I make use of the future verbal tense when I am explaining consequences of the algorithm's instructions that are reflected only after various recursions of the flowchart.

11. This instruction realizes the intention of the previous decision. The rests identified in the previous instruction are substituted by extra attacks (the pitches are equal to the pedal's pitch).
12. This decision sends the algorithm to a recursion if the series from where the section is being constructed has not been completely used yet.
13. If the previous decision was positive, then the algorithm is sent into a recursion that not only considers a linear construction, but that also affects some of the written notes after the last bar displaying a time signature change.
6. This instruction becomes necessary because a time signature is being inserted where pitches had already been organized in time. The instruction calls for such time structure to be preserved while the new metric structure is inserted.
14. This time signature change, the last one in the section, points out that the process, which generated the metric structure, has been finished. The choosing of 4/4 as the new time signature makes reference to: a) a metric ratio that breaks with the eighth-note as common denominator and b) the time signature where most of the music that does not have an emphasis on metric structure is written.
15. This is the same instruction as number 6. It serves the same function.
16. This instruction makes the first note of the section dynamic marking ***pp***
17. By placing a ***f*** dynamic (the loudest dynamic in the section) on the first attack of each time signature, I make an emphasis on the section's metric structure.
18. This instruction constructs a wave-like movement at the dynamic level. The crests of it are at the beginning of each bar and the valleys at the middle of the bar.

Before explaining the construction of the next sections, I find it relevant to make a link between what I said under the subtitle "The Conceptual Material" and the presented algorithm. I make such a parenthesis since it is this first section of the piece that was directly inspired on the conceptual material. The rest of the piece, as previously stated, is more closely linked to this section than to the conceptual material (although, of course, through their bond to this section, they remain linked to the extra musical motivation).

As it is already obvious, the re-reading concept of Donatoni is not applied in this piece. However, I did borrow the concept of applying a sole reiterative algorithm to derive the section; the main difference being that he (Donatoni) applies his flow function to a previous piece, while I do so to abstract pitch material.

Since most fractals are defined through iterative processes, it made sense to apply such a procedure in my piece. In the present case, it was Koch's Curve that inspired the construction of the flowchart.[37] In particular, I borrowed the idea of superimposing structurally equivalent parts, which appear at different scales. This is the core idea behind the algorithm's construction. I consider it relevant to show how this is reflected in my flowchart.

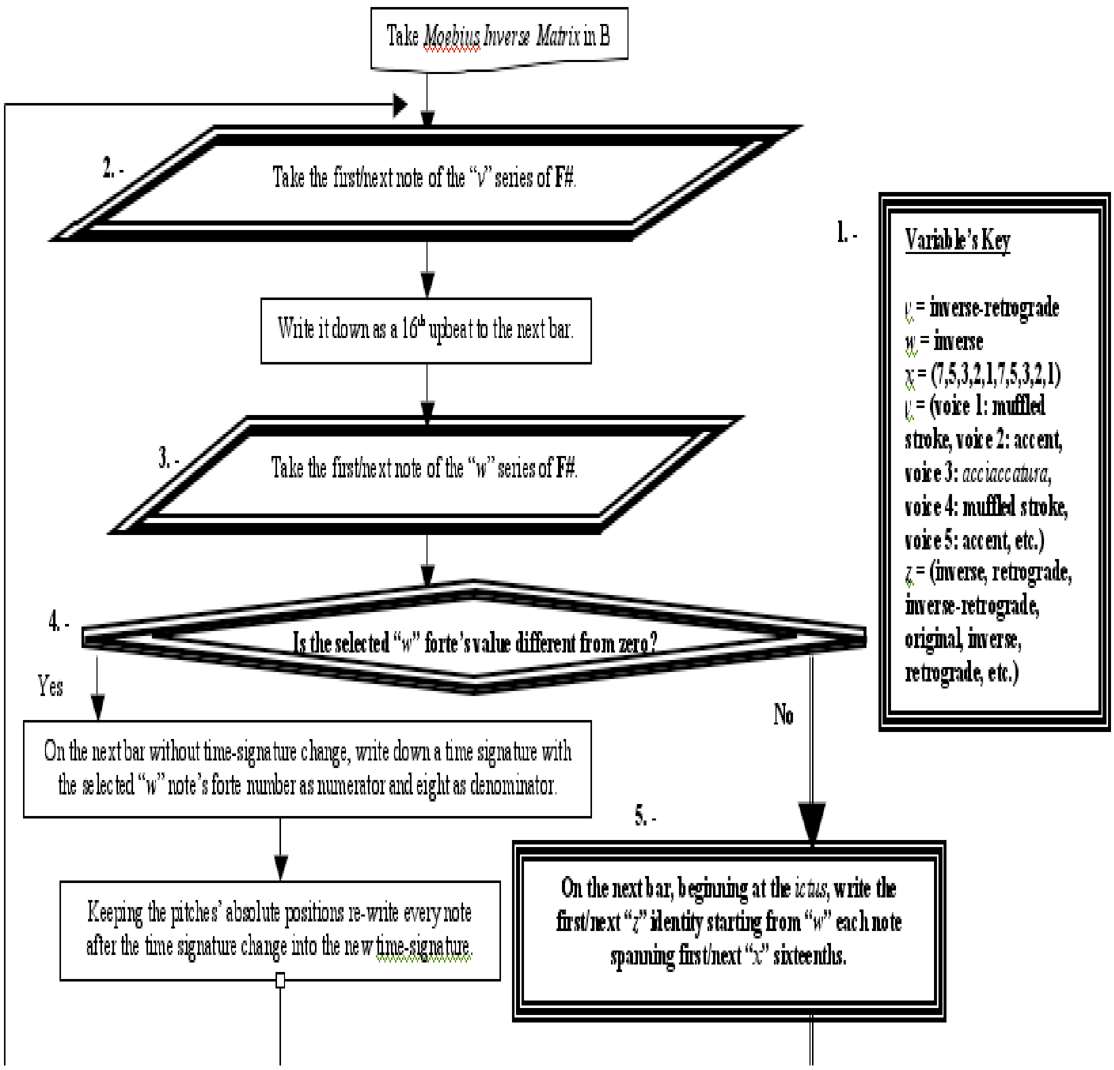
I will begin with the difference of scale. This difference applied to structurally equivalent parts is introduced on instruction number seven where, at every recursion of the building process, a differently scaled ten-pulse isorhythmic pedal is introduced (through instruction thirteen, each sequence starts on the beginning of each new bar). The existence of such differently scaled successions is made perceivable by their occurrence on a different pitch of the Moebius Progression. Instruction eleven is also related to the employment of different scales, this time at the metric level (this instruction also justifies including instruction five inside the recurrent part of the algorithm instead of as an *a posteriori* procedure). By means of adding pulses at the beginning of each bar, but only within the voice being constructed on the relevant recursion, a trace of the constant rewriting of time signatures is preserved.

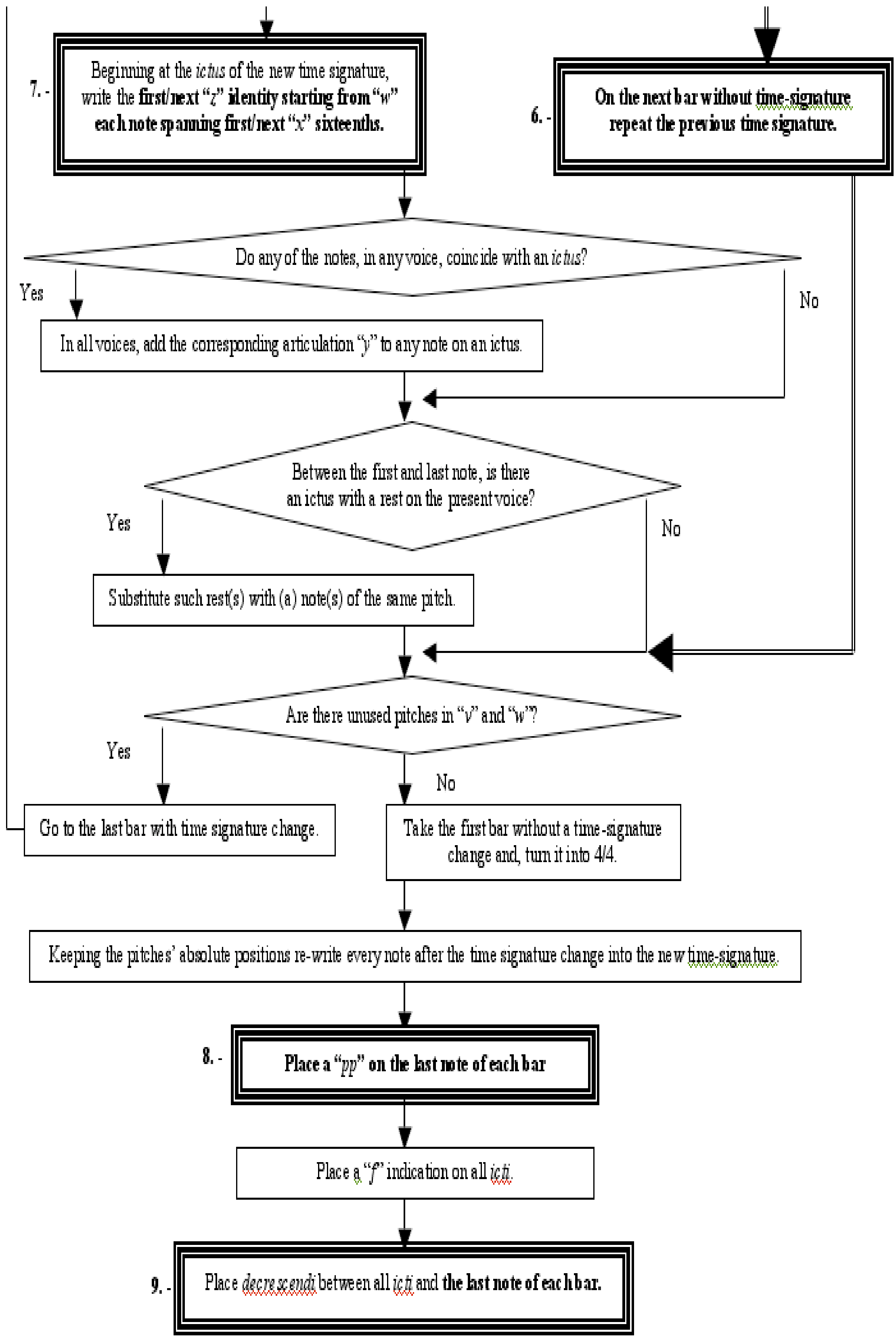
The last idea I took from the relevant fractal was to add an extra sense of structural unity so that not only the different parts would be perceivable but also a relationship between them would affect the piece's behaviour. This is reflected on instruction nine. Here, instructions for adding articulations are given. As opposed to the previously discussed commands, where variety is sought, here, only three different articulations can be used. This is reflected on a sense of uniformity with respect to accents, dead strokes and *acciaccature*. Moreover, such articulations are to be put on the points where both the voice being constructed and the previously created voices coincide with the time signature that corresponds to the recursion in progress.

[37] For a rendering of how such a fractal is constructed, see chapter II.

The second and fourth sections of the piece –between rehearsal marks "A" and "B" and between rehearsals "C" and "D"– were actually composed after the piece was finished (in fact, they do not exist in the original version). They were added to make a clearer separation between the principal sections; a decision made for formal clarity's sake, and will not be explained in this text. They can be seen, paraphrasing a previously made observation on Classical Greek architecture, as the constructional deviations that give a more harmonious formal perception.

On the other hand, the section spanning from rehearsal "B" to rehearsal "C" is closely linked to the beginning of the piece. It was derived from the following algorithm:





As can be seen, the algorithm is quite similar to the previous one. In fact, except for the boxes that have a more elaborated frame, they are identical. As I previously stated, once I transmuted my concept into music, I remained faithful to the latter. This is reflected in the subsequent algorithms. This is the reason why now, since I employed the Moebius Progression under a serial-like procedure, the development follows a similar path. This will become clear with the explanation of the changes:

1. Concerning the changes in the "Variable's Key", the changes obey three natures. The first one is a pitch-oriented variant: while in the first algorithm I was reading the original and retrograde of B, in this variation I read the inverse and inverse-retrograde of F#. The second one obeys a change in the different scales of the isorhythms: now such changes occur at a prime relationship. The articulation variable is also changed. Finally, a new variable is introduced. The reasons for these changes will be made explicit when the relevant instructions appear.
2. This instruction calls for a different identity. The change is subordinated to the main progression that constructs the section; just as in the previous algorithm, the series is the principal series' retrograde.
3. While in the first algorithm the principal series was the top horizontal series (B original), in this variation, I read the rightmost vertical one (F# inverse). As may be already intuited, the adding of the principal series throughout the piece will describe a clockwise reading of the outer edge of the transposition matrix.
4. This binary decision was introduced to go around a problem that surged with respect to the transmutation of the pitch Forte representations into time signatures: in the original series, pitch zero (C) did not exist. Since a metric ratio of 0/8 is undefined, a different procedure needed to be followed given that case.
5. The instruction, together with the next one, deal with the problem mentioned in the previous explanation. It is analogue to the seventh instruction of the first algorithm. However, this time a new variable, "*z*" is introduced. This variable converts the isorhythmic pedals of the previous section into progressions following the transposed identities of the original progression. This development is both a common practice in the serial idiom, and also follows the idea of similarity across scales observed in fractals. Note how, contrary to the flow observed when the previous decision turns out positive,

the writing of the rhythmic structure occurs without a preceding change of time signature.

6. The command gives account of the inexistence of change with respect to metric ratio by allowing two bars with the same time signature. Note how this instruction skips both: 1) the process of re-writing observed when the time signature is instructed to be changed, and 2) the process of adding articulations.
7. This instruction is the same as the one explained in point five, but for the case when the decision treated on point four turns out positive.
8. Makes the last note of each bar ***pp*** (the softest dynamic in the section).
9. Finally, a *decrescendo* between each ictus and the last bar is asked to be plotted; still putting emphasis on the metric changes, but this time in a *sùbito* manner.

The next algorithm-constructed section (between rehearsals "D" and "E") is so similar to the previous one, that I find it pointless to reproduce it. Instead of that, I will directly speak about their differences:

1. The principal progression is now, continuing with a clockwise reading of the outer edge of the transposition matrix, the retrograde of E (it's complementary is now, of course, E original).
2. The scale relationship between the isorhythms is now the complement of the prime relation present in the previous algorithm; that is, the non-prime numbers between one and ten.
3. The reading of the "*z*" variable is analogue to its predecessor but this time the sequence is bended when it reaches the matrix's inversion axis (for instance, D original would become: D, C#, B, Eb, C, F, E, D, Bb, C#).
4. The metric structure is emphasised by repeating the following dynamic pattern: ***f*** (on the *icti*), ***pp*** (on the second note of each bar), *crescendo* (from the second note of each bar until the next bar).

The last section of the piece, from rehearsal "E" until the end, is derived from an algorithm that is almost identical to the first one. The differences are:

1. The last principal progression –the inverse-retrograde of B is employed.
2. The zero present in the above mentioned progression is interpreted as a placement of ten *acciaccature* on ten consecutive notes (parting from where the isorhythmic sequence would start).
3. The isorythmic voices are scaled in accordance to the Forte representation of the pitch they repeat.
4. The emphasis of the metric structure is achieved through an ***f*** on all *icti* and a ***pp*** on the second note of each bar.

Now that the piece's construction has been schematically explained, a few words on how such an abstract construction was instrumented for two marimbas are needed. The first version of the piece was actually thought of for only one marimba. The idea was to find a way to broaden the relatively closed idiomatic treatment of the instrument. I thought that an interesting way of doing so was to explore the common technique of holding two mallets per hand in order to develop a, mostly rhythmical, polyphonic approach to the instrument.

To make such a concept perceivable, I decided to ask the performer to hold four different kinds of mallets at the same time. The nature of such a technique pretty much organized, within each algorithmically derived section, the register placement of the lines I had come up with. In fact, some extra adapting was needed: some voices were unified since, at some points, there are up to six voices sounding at the same time. This contrasted against the possibilities of timbre variation offered by only four different kinds of mallets (using three mallets per hand was discarded due to the technical complexity already offered by the four mallet option).

Outside of that, I arranged the register displacement that resulted from the four algorithms so that the four sections would make sense as a whole, and to balance the tendency that the development of the sections has towards register uniformity.

Finally, an account on how the music migrated from a single instrument piece into the two-marimba version is due. The soloist version was presented at a "Composing for the Marimba" workshop organized by Peter Prommel. Although the piece was at no moment questioned as unplayable, only part of it could be performed due to the technical and conceptual difficulties it presented within the limited time budget. Peter Prommel, who was providing feedback to both percussionists and composers, commented that the musical idea behind the piece was very interesting but that it sounded in his mind as duo music. It was thanks to such a criticism on the one hand (a comment that, I must confess, I had already received from Wim Henderickx), and thanks to the support of my musical idea, on the other hand, that I decided to make the two-marimba version.

V. **Conclusion**

In the introductory chapter of the present text, a formalization process of an extra musical motivation for composing –Chaos Theory– was proposed. Such a method followed two separate paths: an analytical one and a theoretical one. Finally an intersection of both approaches was made through the compositional practice.

The analyzed music was Franco Donatoni's *Françoise Variationen I, II* and *III*. The purpose of chapter II was to use the concept of fractal as a means to look into an aspect of Donatoni's music. While, from the beginning, it was stated that the variations were not fractals, the analogy was made in order to look at how the re-readings of Donatoni preserve cross-scale structural similarity. The analysis looked into the variations' structural similarities. First, I exposed how the different movements were developed from their respective predecessor by means of different re-reading logics. Then, a cross-scaled similarity between the structures of the different variations was established. Finally, a relationship between the formal nature of the re-read variation and its effect on the structural nature of its descendant was drawn. There were several aspects relevant to this analytical approach that were left out. For instance, trying to establish a connection between the different re-reading logics.

The idea of a sole reading logic developing a musical movement is a technical aspect that was inspired by the analysis of the variations. In the case of my compositional work, the idea was to create similar reading logics and apply them to a sole abstract material. In this way, I wanted to organize my music following the idea of an interaction between differently scaled and developed expressions of my abstract material. This was inspired by the analogy that I have already established between the family and society.[38]

As to my abstract material, chapter III deals with its construction. The material was constructed through an iterative process. The goal was to use this potentially chaotic procedure in order to generate a system that exhibited chaotic behaviour. The main difficulty of achieving such a task was the fact that the Chromatic Set is an extremely regular one. The problem was approached by eliminating one note and constructing the system on a base 11 modular system (instead of within the base 12 chromatic scale). A simple function that described how the pitch development behaved

[38] See page 41.

was iterated. The mentioned function's –modifiable– initial conditions were defined (starting note and missing pitch). Then, an exploration of how the possible combinations of pairs of initial conditions behaved along the iterations was made through a statistic qualitative approach. The idea was to try to determine if the system's overall behaviour exhibited chaotic behaviour. The system exhibited both irregular and uniform behaviours. The usability of the system as a means to obtain a chaotic-like behaviour was limited to the choosing of the initial conditions to be employed.

The relationship between this material and *Mechanism for Two Marimbas* is clear. The reason I chose to use only the identities of the Moebius Progression as material is also linked to the family-society association. I was interested in working with differently scaled elements that shared a common structural nature. The interaction between such elements was dictated by an independent algorithm but following parameters dictated by the same structural characteristics of the employed progression.

As a result of the articulation between my reading approach and my abstract material, *Mechanism for two Marimbas* was conceived. Since I am writing a text under academic circumstances, I have explained my creative process. However, as an artist, I do not consider it ethical to make an interpretation of my music, as that would imply a judgement, which I think the listener must make for him or her self. Furthermore, I am not interested in trying to project my contextual material as the fixed content of the music (a possibility that remains questionable). On the contrary, my interest is in providing structurally congruent musical forms that, while suggesting a kind of organizational logic, appeal to the listener's experience to make meaning out of them.

Finally, concerning my regards on the formalization process, I can say a couple of things. The first one is that affirming that the present work is an account of a possible formalization process of Chaos Theory as an approach to composition would be going too far. Nonetheless, it does set certain grounds for such a task. Secondly, I believe that the present exploration of such formalization has already had positive effects in the development of my craftsmanship. This, I think, has been reflected in the on-going process of maturing and consolidating my compositional language. In addition, ideas for further theoretical and practical (i.e. compositional) experimentation have been generated.

Appendix A: Resulting progression and looping section of all possible initial conditions

Note Taken Away	Resulting Progression										Progression's Loop Section
C (0)[*]	1	2	4	8	5	10	9	7	3	6	Whole progression
	2	4	8	5	10	9	7	3	6	1	Whole progression
	3	6	1	2	4	8	5	10	9	7	Whole progression
	4	8	5	10	9	7	3	6	1	2	Whole progression
	5	10	9	7	3	6	1	2	4	8	Whole progression
	6	1	2	4	8	5	10	9	7	3	Whole progression
	7	3	6	1	2	4	8	5	10	9	Whole progression
	8	5	10	9	7	3	6	1	2	4	Whole progression
	9	7	3	6	1	2	4	8	5	10	Whole progression
	10	9	7	3	6	1	2	4	8	5	Whole progression
	11										11
C# (1)	0										0
	2	4	8	5	10	9	7	3	6	0	0
	3	6	0								0
	4	8	5	10	9	7	3	6	0		0
	5	10	9	7	3	6	0				0
	6	0									0
	7	3	6	0							0
	8	5	10	9	7	3	6	0			0
	9	7	3	6	0						0
	10	9	7	3	6	0					0
	11										11
D (2)	0										0
	1	3	6	0							0
	3	6	0								0
	4	8	5	10	9	7	3	6	0		0
	5	10	9	7	3	6	0				0
	6	0									0
	7	3	6	0							0

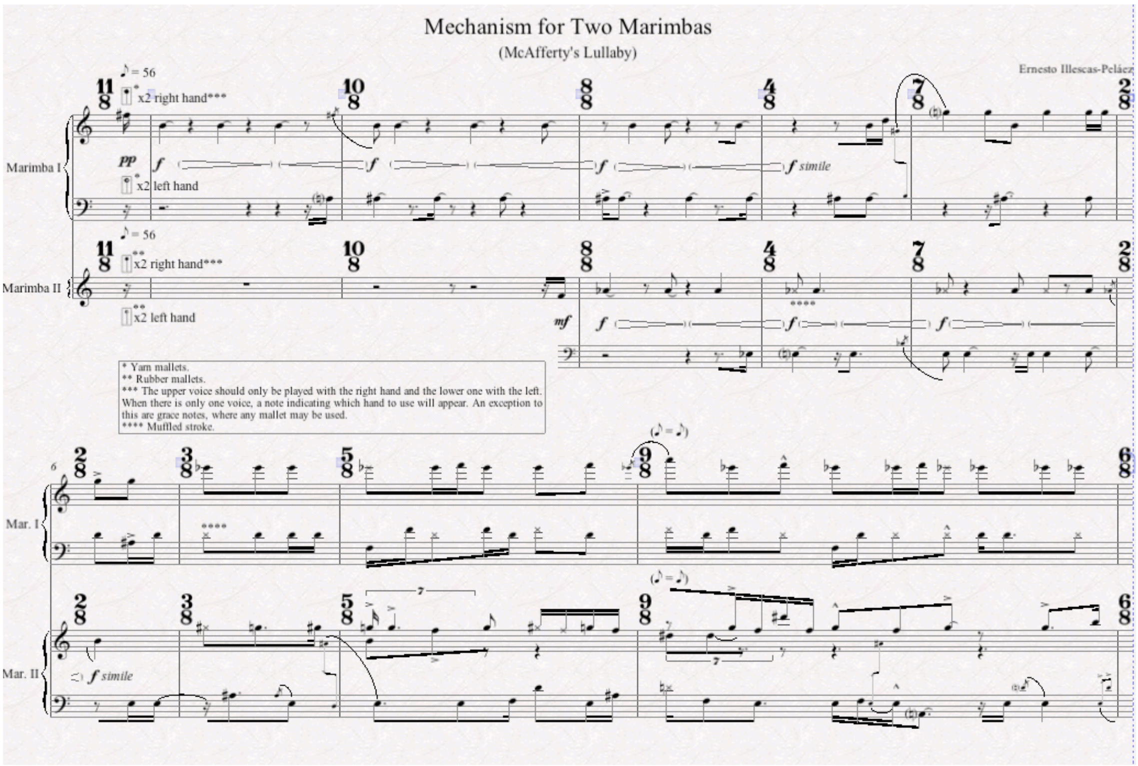
[*] Original

	8	5	10	9	7	3	6	0			0				
	9	7	3	6	0						0				
	10	9	7	3	6	0					0				
	11										11				
Eb (3)	0										0				
	1	2	5	10	9	7					2	5	10	9	7
	2	5	10	9	7						Whole progression				
	4	8	5	10	9	7	2				5	10	9	7	2
	5	10	9	7	2						Whole progression				
	6	0									0				
	7	2	5	10	9						Whole progression				
	8	5	10	9	7	2					5	10	9	7	2
	9	7	2	5	10						Whole progression				
	10	9	7	2	5						Whole progression				
	11										11				
E (4)	0										0				
	1	2	5	10	9	7					2	5	10	9	7
	2	5	10	9	7						Whole progression				
	3	7	2	5	10	9					7	2	5	10	9
	5	10	9	7	2						Whole progression				
	6	0									0				
	7	2	5	10	9						Whole progression				
	8	5	10	9	7	2					5	10	9	7	2
	9	7	2	5	10						Whole progression				
	10	9	7	2	5						Whole progression				
	11										11				
F (5)	0										0				
	1	2	4	9	7						2	4	9	7	
	2	4	9	7							Whole progression				
	3	7	2	4	9						7	2	4	9	
	4	9	7	2							Whole progression				
	6	0									0				
	7	2	4	9							Whole progression				
	8	4	9	7	2						4	9	7	2	
	9	7	2	4							Whole progression				

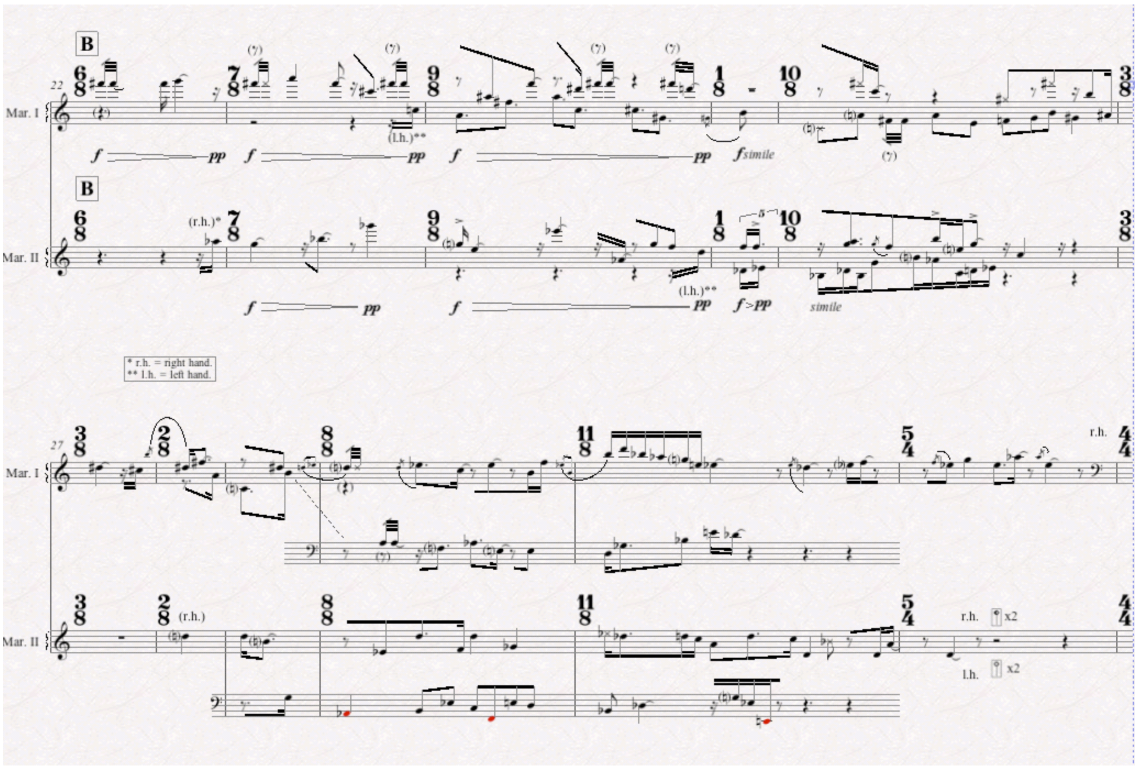
	10	9	7	2	4						9	7	2	4	
	11										11				
F# (6)	0										0				
	1	2	4	9	7						2	4	9	7	
	2	4	9	7							Whole progression				
	3	7	2	4	9						7	2	4	9	
	4	9	7	2							Whole progression				
	5	11									11				
	7	2	4	9							Whole progression				
	8	4	9	7	2						4	9	7	2	
	9	7	2	4							Whole progression				
	10	9	7	2	4						9	7	2	4	
	11										11				
G (7)	0										0				
	1	2	4	9	6						Whole progression				
	2	4	9	6	1						Whole progression				
	3	6	1	2	4	9					6	1	2	4	9
	4	9	6	1	2						Whole progression				
	5	11									11				
	6	1	2	4	9						Whole progression				
	8	4	9	6	1	2					4	9	6	1	2
	9	6	1	2	4						Whole progression				
	10	9	6	1	2	4					9	6	1	2	4
	11										11				
G# (8)	0										0				
	1	2	4	9	6						Whole progression				
	2	4	9	6	1						Whole progression				
	3	6	1	2	4	9					6	1	2	4	9
	4	9	6	1	2						Whole progression				
	5	11									11				
	6	1	2	4	9						Whole progression				
	7	3	6	1	2	4	9				6	1	2	4	9
	9	6	1	2	4						Whole progression				
	10	9	6	1	2	4					9	6	1	2	4
	11										11				

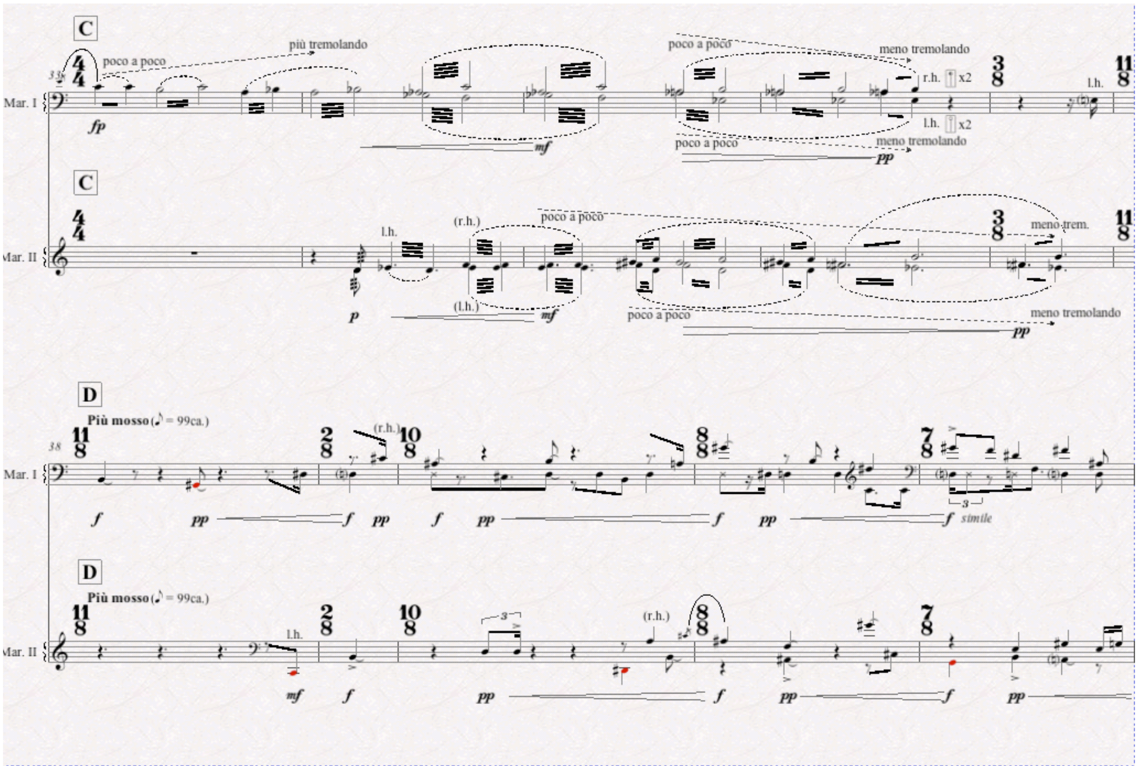
A (9)	0										0
	1	2	4	8	5	11					11
	2	4	8	5	11						11
	3	6	1	2	4	8	5	11			11
	4	8	5	11							11
	5	11									11
	6	1	2	4	8	5	11				11
	7	3	6	1	2	4	8	5	11		11
	8	5	11								11
	10	8	5	11							11
	11										11
Bb (10)	0										0
	1	2	4	8	5	11					11
	2	4	8	5	11						11
	3	6	1	2	4	8	5	11			11
	4	8	5	11							11
	5	11									11
	6	1	2	4	8	5	11				11
	7	3	6	1	2	4	8	5	11		11
	8	5	11								11
	9	7	3	6	1	2	4	8	5	11	11
	11										11
B (11)	0										
	1	2	4	8	5	10	9	7	3	6	Whole progression
	2	4	8	5	10	9	7	3	6	1	Whole progression
	3	6	1	2	4	8	5	10	9	7	Whole progression
	4	8	5	10	9	7	3	6	1	2	Whole progression
	5	10	9	7	3	6	1	2	4	8	Whole progression
	6	1	2	4	8	5	10	9	7	3	Whole progression
	7	3	6	1	2	4	8	5	10	9	Whole progression
	8	5	10	9	7	3	6	1	2	4	Whole progression
	9	7	3	6	1	2	4	8	5	10	Whole progression
	10	9	7	3	6	1	2	4	8	5	Whole progression

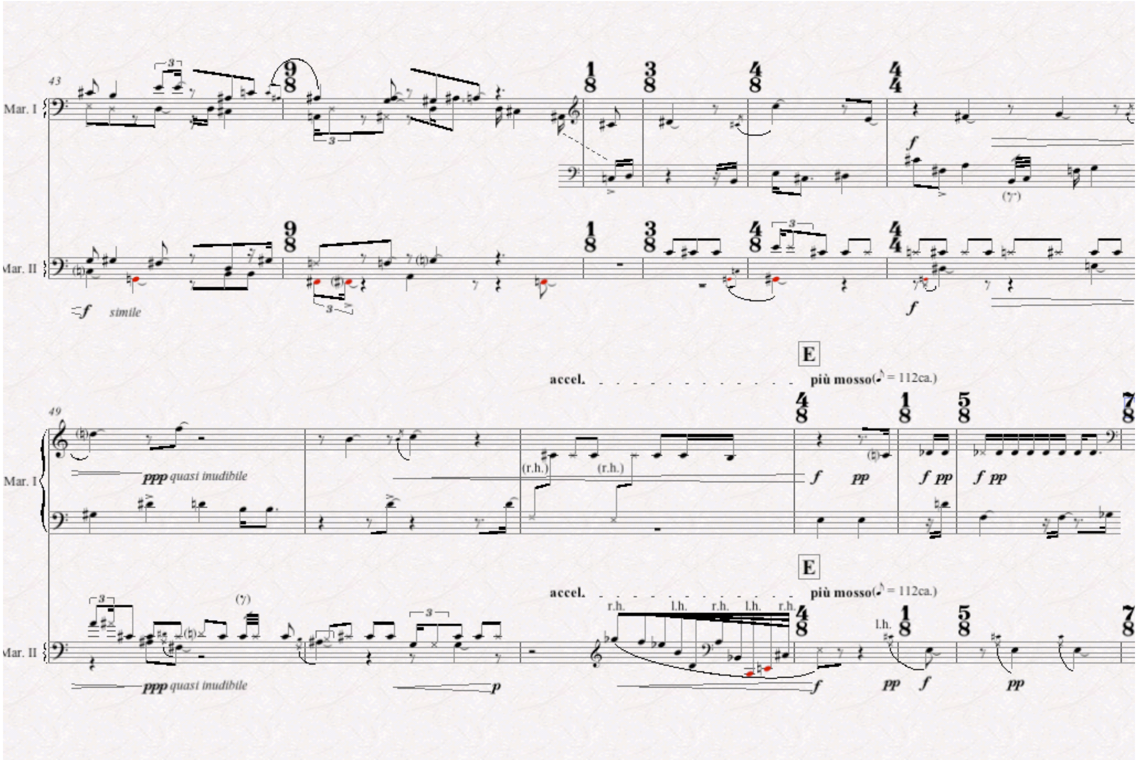
Appendix B


Mechanism for Two Marimbas
(McAfferty's Lullaby)
Ernesto Illescas-Peláez
♪ = 56
x2 right hand***
x2 left hand
Marimba I
Marimba II
f simile
* Yarn mallets.
** Rubber mallets.
*** The upper voice should only be played with the right hand and the lower one with the left. When there is only one voice, a note indicating which hand to use will appear. An exception to this are grace notes, where any mallet may be used.
**** Muffled stroke.
Mar. I
Mar. II


accel.
più mosso (♩ = 75ca.)
ppp quasi inudibile
Mar. I
Mar. II
A
(r.h.)*
* r.h. = right hand


B
Mar. I
Mar. II
(l.h.)**
(r.h.)*
f simile
f>pp
simile
* r.h. = right hand.
** l.h. = left hand.
(r.h.)
r.h.
l.h.
x2


C
poco a poco
più tremolando
meno tremolando
r.h.
l.h.
x2
(r.h.)
(l.h.)
meno trem.
D
Più mosso (♪ = 99ca.)
Mar. I
Mar. II
simile


43
Mar. I
Mar. II
simile
49
E
accel.
più mosso(♪ = 112ca.)
ppp quasi inudibile
(r.h.)
r.h.
l.h.


55
Mar. I
Mar. II
simile
(l.h.)
(r.h.)
59
63
r.h.
l.h.
Durata ca. 5' 30''